Proxy Advisory Firms: The Economics of Selling Information to Voters

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Abstract

We analyze how proxy advisors, which sell voting recommendations to shareholders, affect corporate decision-making. If the quality of the advisor's information is low, there is overreliance on its recommendations and insufficient private information production. In contrast, if the advisor's information is precise, it may be underused because the advisor rations its recommendations to maximize profits. Overall, the advisor's presence leads to more informative voting only if its information is sufficiently precise. We evaluate several proposals on regulating proxy advisors and show that some suggested policies, such as reducing proxy advisors' market power or decreasing litigation pressure, can have negative effects.

1 Introduction

Proxy advisory firms provide shareholders with research and recommendations on how to cast their votes at shareholder meetings of public companies. For highly diversified institutional investors, the costs of performing independent research on each issue on the agenda in each of their portfolio companies are substantial. The institution may prefer to pay a fee and get information from a proxy advisory firm instead. In the last years, the demand for proxy advisory services has substantially increased due to several factors – the rise in institutional ownership, the 2003 SEC rule requiring mutual funds to vote in their clients' best interests, and the growing volume and complexity of issues voted upon. The largest proxy advisor, Institutional Shareholder Services (ISS), has over 1,900 institutional clients and covers about 40,000 meetings around the world. By now, there is strong empirical evidence that proxy advisors' recommendations have a large influence on voting outcomes.[1] This influence has attracted the attention of the SEC and regulatory bodies in other countries and has led to a number of proposals to regulate the proxy advisory industry.

Market participants concerned about the influence of proxy advisors often emphasize potential deficiencies in their recommendations – the one-size-fits-all approach to governance, inaccuracies in their methodologies and data, and potential conflicts of interest.[2] Other observers counter that even if the quality of proxy advisors' recommendations is not high, market forces will ensure that information production and aggregation in voting will be efficient because "institutional investors are sophisticated market participants that are free to choose whether and how to employ proxy advisory firms" (GAO report, 2016). According to Nell Minow, a well-known governance expert, "what we have is the most sophisticated institutional investors in the world...making a free market decision to pay for outside, objective analysis...There could not be a better example of market efficiency."[3]

In this paper, we emphasize that the market efficiency view does not take into account the collective action problem among shareholders. We show that because shareholders do not internalize the effect of their actions on other shareholders, there may be excessive overreliance on proxy advisors' recommendations and, as a result, excessive conformity in shareholders' votes. Moreover, this problem might not be resolved by improving the quality

[1] See Alexander et al. (2010), Ertimur, Ferri, and Oesch (2013), Iliev and Lowry (2015), Larcker, McCall, and Ormazabal (2015), Malenko and Shen (2016), and McCahery, Sautner, and Starks (2016), among others.

[2] See, e.g., Gallagher (2014) and the SEC 2010 Concept Release on the U.S. Proxy System.

[3] Harvard Law School Forum on Corporate Governance, March 2 2018.

of proxy advisors' recommendations.

Specifically, the goal of our paper is to provide a simple framework for analyzing the economics of the proxy advisory industry. We are particularly interested in understanding how proxy advisors affect the quality of corporate decision-making and in analyzing the effects of the suggested policy proposals. For this purpose, we build a model of strategic voting in the presence of a proxy advisory firm. In the model, shareholders are voting on a proposal that can increase or decrease firm value with equal probability. Each shareholder can acquire information about the value of the proposal from two sources – do his own independent research or get information from the proxy advisor. For example, in practice, some institutions have their own proxy research departments, while others strongly rely on proxy advisors' recommendations.[4] More specifically, there is a monopolistic proxy advisor that has an informative signal about the proposal. The advisor sets a fee that maximizes its profits and offers to sell its signal to shareholders for this fee. Each shareholder then independently decides whether to buy the advisor's signal, to pay a cost to acquire his own signal, to acquire both signals, or to remain uninformed. After observing the signals he acquired, each shareholder decides how to vote, and the proposal is implemented if it is approved by the majority of shareholders.

In this framework, the proxy advisor provides a valuable service: an option to buy and follow an informative signal. The presence of this option, however, comes at a cost: it reduces shareholders' incentive to invest in their own independent research. If the firm were owned by a single shareholder, he would perfectly internalize the effect of his decisions on firm value and would choose between the two sources of information efficiently. However, the firm is owned by multiple shareholders, leading to a collective action problem and inefficiencies in information acquisition. Specifically, a shareholder who acquires information (privately or from the proxy advisor) imposes a positive externality on other shareholders by making the vote more informed. When some other shareholders already follow the proxy advisor, this externality is higher if a shareholder acquires information privately than if he acquires information from the advisor. This is because when shareholders follow their private signals, they make independent (or, more generally, imperfectly correlated) mistakes. In contrast,

[4] Iliev and Lowry (2015) show that there is substantial heterogeneity among institutions in the extent to which they rely on ISS, and Iliev, Kalodimos, and Lowry (2018) show that institutions doing more independent governance research (as measured by their views of firms' proxy-related SEC filings) are significantly less likely to vote with ISS. See also Ertimur, Ferri, and Oesch (2013), Larcker, McCall, and Ormazabal (2015), and Malenko and Shen (2016).

when shareholders follow the same signal (advisor's recommendation), their mistakes are perfectly correlated, which increases the probability that an incorrect decision will be made. Therefore, the collective action problem may lead to excessive overreliance on the advisor's recommendations and crowd out too much private information production.

This trade-off between providing a new informative signal, on the one hand, and crowding out independent research and generating correlated mistakes in votes, on the other hand, leads to our main result: The presence of the proxy advisor increases firm value (the probability of a correct decision being made) only if the precision of its recommendations is sufficiently high. This result holds for any cost of the proxy advisor's recommendations, even if this cost is very small.[5]

The fact that the proxy advisor sets its fee strategically, aiming to maximize its own profits rather than the informativeness of voting, creates inefficiency of another sort. In our model, when the advisor's information is imprecise, firm value would be maximized if its recommendations could be made prohibitively costly, to maximize shareholders' incentives to invest in independent research. In contrast, when the advisor's information is sufficiently precise, firm value would be maximized if the price of its recommendations could be made as low as possible. Clearly, neither of these policies corresponds to what the monopolistic proxy advisor finds optimal to do. When the advisor's information is imprecise, it charges low fees to induce shareholders to buy its recommendations. This crowds out independent research and leads to overreliance on the advisor's recommendations. In contrast, when the advisor's information is very precise, it becomes underused: to maximize profits, the monopolistic advisor rations it and sells it to only a fraction of investors. Interestingly, because of this strategic pricing, informativeness of voting sometimes goes down even if the advisor's information is perfectly precise, as long as the quality of decision-making without the advisor is sufficiently high.

The firm's ownership structure plays a key role for whether the advisor's presence has a positive effect on the informativeness of voting. In firms with highly dispersed ownership, the collective action problem is so severe that if the proxy advisors' recommendations were not available, there would be very little private information production and voting would be uninformative. In such firms, the negative crowding out effect does not arise, while the

[5]For example, anecdotal evidence suggests that a large fraction of institutions subscribe to either ISS or Glass Lewis, implying that in practice, the proxy advisors' fees are not high. In addition, proxy advisors' recommendations are sometimes made public in high profile cases. See also the discussion in Section 7.

positive effect does – the advisor's presence provides a relatively cheap way for shareholders to become informed. Thus, the negative effect of proxy advisors is more likely to arise in firms with more concentrated ownership, which is consistent with the findings of Calluzzo and Dudley (2017).

In our basic model, the only reason shareholders subscribe to the proxy advisor is to make more informed voting decisions. Another frequently discussed motive for following proxy advisors' recommendations is that it could protect an institutional investor from potential litigation.[6] We therefore introduce the risk of litigation for shareholders' voting decisions, which shareholders can eliminate by following the advisor's recommendations. The positive effect of greater litigation pressure is that it increases shareholders' incentives to vote informatively. However, greater litigation pressure may also exacerbate the crowding out effect by inducing shareholders to follow the advisor instead of doing independent research. As a result, we show that greater litigation pressure decreases the informativeness of shareholder voting if the advisor's recommendations are not of high enough quality.

Finally, we use the model to evaluate the frequently discussed proposals on regulating the proxy advisory industry. They include reducing the market power of ISS and Glass Lewis to lower the costs of proxy advisory services (GAO, 2007), improving the quality of proxy advisors' recommendations, and increasing the transparency about their data, methodologies, and conflicts of interest (Edelman, 2013). We show that decreasing the advisor's fees has a positive effect on firm value if its recommendations are of high enough quality, but it could have an unintended negative effect if the quality of recommendations is low: in this case, lowering the fees would encourage even more investors to follow the imprecise advisor's recommendations instead of doing independent research. Similarly, both improving the quality of the advisor's recommendations and increasing the transparency about its methodologies and conflicts of interest can have either a positive and negative effect, depending on how precise its recommendations are. Overall, our results suggest that any regulation of proxy advisors should carefully take into account how it will affect private information acquisition by investors and how informative proxy advisors' recommendations are.

The paper proceeds as follows. The remainder of this section reviews the literature.

[6] As the former SEC commissioner Daniel M. Gallagher put it, "relying on the advice from the proxy advisory firm became a cheap litigation insurance policy: for the price of purchasing the proxy advisory firm's recommendations, an investment adviser could ward off potential litigation over its conflicts of interest" (Gallagher, 2014). Indeed, the 2003 SEC rule and the two 2004 SEC no-action letters discussed in Section 8 suggest that following the recommendations of a proxy advisor is a means of ensuring that an institutional investor satisfies its fiduciary duty to vote in its clients' best interests.

Section 2 describes the setup and solves for the benchmark case without a proxy advisor. Section 3 analyzes shareholders' information acquisition and voting decisions in the presence of a proxy advisor and derives implications for the quality of decision-making. Section 4 discusses the advisor's pricing strategy. Section 5 analyzes litigation pressure and several policy proposals. Section 6 extends the model by endogenizing the quality of the advisor's recommendation, and Section 7 discusses other possible extensions of the basic model. Section 8 outlines the empirical implications. Finally, Section 9 concludes.

Related literature

Our paper is related to papers that study voting in the corporate finance context. Maug (1999) and Maug and Yilmaz (2002) examine conflicts of interest between voters, Bond and Eraslan (2010) study voting on an endogenous agenda in the debt restructuring context (among other contexts), Brav and Mathews (2011) analyze empty voting, Levit and Malenko (2011) study nonbinding voting on shareholder proposals, and Van Wesep (2014) proposes a voting mechanism that would increase shareholder turnout. Our paper contributes to this literature by analyzing another important institutional feature of corporate voting – the presence of proxy advisors.

More generally, our paper is related to the literature on strategic voting in economics, which studies how information that is dispersed among voters is aggregated in the vote (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998). It is mostly related to papers that analyze endogenous information acquisition by voters (Persico, 2004; Martinelli, 2006; Gerardi and Yariv, 2008; Gershkov and Szentes, 2009; Khanna and Schroder, 2015). Differently from these papers, which focus on how the number of voters and the decision-making rule affect information acquisition and the quality of voting, our focus is on the effect of information sales by a third party. Alonso and Camara (2016), Chakraborty and Harbaugh (2010), Jackson and Tan (2013), and Schnakenberg (2015) analyze information provision by biased senders to voters, in the form of either communication or Bayesian persuasion. Their focus is on how the sender exploits heterogeneity in voters' preferences to sway the outcome in his favor, while our model features no conflicts of interest between parties and instead focuses on the sale of information and crowding out of private information acquisition.

The fact that the proxy advisor sells its information relates our paper to the literature on the sale of information. It includes literature on selling information to traders in financial markets (e.g., Admati and Pfleiderer, 1986, 1990; Fishman and Hagerty, 1995; Cespa, 2008; and Garcia and Sangiorgi, 2011, among others), as well as information sales in other contexts (e.g., Bergemann, Bonatti, and Smolin, 2017). To our knowledge, our paper is the first to study the sale of information to agents who can also engage in private information acquisition. Our second contribution is to examine information sales in a strategic voting context. There are two important differences between selling information to voters and traders, which make our setting and results different from those in the literature: first, voters have common interests while traders compete with each other; second, in voting, a voter cares about the event in which he is pivotal.

Finally, on a broader level, our paper relates to a large literature on externalities in information acquisition and aggregation. This literature includes papers that examine how public information disclosure affects investors' incentives for private information production (e.g., Diamond, 1985; Boot and Thakor, 2001; Piccolo and Shapiro, 2017) and use (e.g., Bond and Goldstein, 2015).[7] It also includes papers that examine inefficiencies in the use of information (Morris and Shin, 2002; Angeletos and Pavan, 2007), acquisition of information (Hellwig and Veldkamp, 2009), or both information acquisition and information use (Colombo, Femminis, and Pavan, 2014) due to payoff externalities among agents, such as strategic complementarity or substitutability between agents' actions.[8] The focus on voting makes our paper quite different from these literatures. The difference from the former literature, where the interplay between public information and private information acquisition and use works through trading profit considerations, is that the mechanism in our paper is through shareholders' beliefs about the effect of their decisions on voting outcomes. Our mechanism is also quite different from the latter literature because in our model, shareholders do not care about coordinating their votes per se: each shareholder only cares about maximizing the value of his shares less the information acquisition costs. In addition, differently from the literature, we focus on the sale of information by a profit-maximizing seller.

[7] In addition, the feature that agents can acquire information directly or via an intermediary (proxy advisor) connects our paper to theories of financial intermediation, such as Diamond (1984) and Ramakrishnan and Thakor (1984).

[8] Information aggregation is also inefficient in herding models but for a different reason – sequential decision-making by agents (e.g., Bikhchandani, Hirshleifer, and Welch, 1992; Banerjee, 1992). Khanna and Mathews (2011) study information acquisition in a herding context.

2 Model setup

We adopt the standard setup in the strategic voting literature (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998) and augment it by introducing an advisor that offers to sell its signal to the voters.

The firm is owned by $N \geq 3$ shareholders, where N is odd. Each shareholder owns the same stake in the firm (for simplicity, one share), and each share provides one vote. It is easiest to think about these shareholders as the company's institutional investors: given their often significant holdings in the companies and their fiduciary duties to their clients, they are likely to have incentives to vote in an informed way and hence to incur the costs of private information acquisition or the costs of buying proxy advisors' recommendations.

There is a proposal to be voted on at the shareholder meeting, which is implemented if it is approved by the majority, i.e., if at least $\frac{N+1}{2}$ shareholders vote for it.[9] Let d denote whether the proposal is accepted ($d = 1$) or rejected ($d = 0$). The value of the proposal, and thus the optimal decision $d^* \in \{0, 1\}$, depends on the unknown state $\theta \in \{0, 1\}$, where both states are equally likely. Without loss of generality, assume that the optimal decision is to match the state, i.e., accept the proposal if $\theta = 1$ and reject it if $\theta = 0$. Specifically, firm value per share increases by one if the proposal is accepted in state $\theta = 1$ and decreases by one if it is accepted in state $\theta = 0$. If the proposal is rejected, firm value does not change. Denoting the change in firm value per share by $u(d, \theta)$,

$$\begin{aligned} u(1,\theta) &= \begin{cases} 1, & \text{if } \theta = 1, \\ -1, & \text{if } \theta = 0, \end{cases} \\ u(0,\theta) &= 0. \end{aligned} \tag{1}$$

For example, the vote could correspond to a proxy contest, where the dissident's effect on firm value is either positive ($\theta = 1$) or negative ($\theta = 0$) and the proposal voted on is whether to approve the dissident's nominees. If $d = 1$ (the dissident wins the contest), firm value increases if and only if $\theta = 1$, while if $d = 0$ (the incumbent management stays in place),

[9] While this formulation assumes that the vote is binding, our setup can also apply to nonbinding votes. First, the 50% voting threshold is an important cutoff, passing which leads to a significantly higher probability of proposal implementation even if the vote is nonbinding (e.g., Ertimur, Ferri, and Stubben, 2010; Cuñat, Gine, and Guadalupe, 2012). Second, Levit and Malenko (2011) show that nonbinding voting is equivalent to binding voting with an endogenously determined voting cutoff that depends on company and proposal characteristics.

firm value is unchanged.[10]

Each shareholder maximizes the value of his share minus any costs of information acquisition (Section 5.1 analyzes an extension in which shareholders are also concerned with litigation for their voting practices). Each shareholder can potentially get access to two signals – his private signal and the recommendation of an advisor (the proxy advisory firm). Specifically, the advisor's information is represented by signal ("recommendation") $r \in \{0, 1\}$, whose precision is given by $\pi \in [\frac{1}{2}, 1]$:

$$\Pr(r = 1|\theta = 1) = \Pr(r = 0|\theta = 0) = \pi. \tag{2}$$

For example, Alexander et al. (2010) provide evidence that ISS recommendations in proxy contests seem to convey substantive information about the contribution of dissidents to firm value. Relatedly, according to the survey of institutional investors conducted by McCahery, Sautner, and Starks (2016), 55% of respondents believe that proxy advisors help them make more informed voting decisions.

Each shareholder can buy the advisor's recommendation for fee f, which is optimally set by the advisor at the initial stage. We assume that the advisor's recommendation is simply given by r, so that a shareholder who subscribes to the advisor's services observes r. In the Online Appendix, we show that the advisor does not benefit from personalizing recommendations by adding i.i.d. noise.[11]

In addition to the advisor's signal, each shareholder has access to a private information acquisition technology, whereby shareholder i can acquire a private signal $s_i \in \{0, 1\}$ at a cost $c > 0$. The precision of the private signal is given by $p \in [\frac{1}{2}, 1]$:

$$\Pr(s_i = 1|\theta = 1) = \Pr(s_i = 0|\theta = 0) = p. \tag{3}$$

All signals are independent conditional on state θ, and precision levels p and π are common knowledge.

The timing of the model is illustrated in Figure 1. There are four stages. At Stage 1, the advisor sets fee f that it charges each shareholder who buys the recommendation. At

[10] Fos (2017) provides evidence that in voted proxy contests, dissidents win in 55% of cases.

[11] In practice, proxy advisors sometimes give personalized vote recommendations to clients that have a strong position on particular issues, e.g., on corporate social responsibility proposals. Such behavior would arise in our model if we assumed that shareholders have heterogeneous preferences, the feature that we abstract from in this paper.

Stage 2, each shareholder independently and simultaneously decides on whether to acquire his private signal at cost c, acquire the advisor's signal for fee f, acquire both signals, or remain uninformed. At Stage 3, each shareholder i privately observes the signals he acquired, if any, and decides on his vote $v_i \in \{0, 1\}$, where $v_i = 1$ ($v_i = 0$) corresponds to voting in favor of (against) the proposal. The votes are cast simultaneously. At Stage 4, the proposal is implemented or not, depending on whether the majority of shareholders voted for it, and the payoffs are realized.

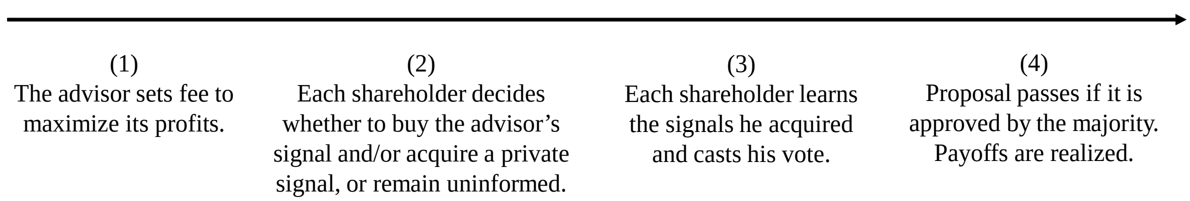

Figure 1. Timeline of the model.

For simplicity, we set up the model in the context of a single proposal. In practice, the decision whether to subscribe to a proxy advisor's recommendations is often made at the investment portfolio level: for example, an institution that subscribes to ISS will receive vote recommendations for each company in its portfolio. Hence, fee f can be interpreted as the fee for a representative firm in the investor's portfolio. Likewise, the decision of whether to pay cost c can be interpreted as the decision of whether to establish a proxy research department. We discuss this "bundling" of proposals by proxy advisors in Section 7.

The setup also assumes that shareholders do not communicate with each other and hence do not observe each others' information when voting. In practice, while some communication between shareholders is possible, the extent of this communication is limited. In particular, there is a fine line between shareholders sharing their information with each other and coordinating with each other. The latter can be viewed as "forming a group" (as defined by the SEC) and requires the filing of Schedule 13D, making shareholders cautious about communicating with each other.[12]

[12] For example, according to the 2011 report by the law firm Dechert LLP, "shareholder concern about unintentionally forming a group has chilled communications among large holders of shares in U.S. public companies." Relatedly, according to the survey of institutional investors by McCahery, Sautner, and Starks (2016), investors believe that "rules on "acting in concert" discourage coordination," and name it as one the most important impediments to shareholder engagement.

We focus on symmetric Bayes-Nash equilibria. Symmetry means two things. First, all shareholders follow the same information acquisition strategy, and at the voting stage, all shareholders of one type (i.e., those who acquired the recommendation from the advisor; those who acquired a private signal; those who acquired neither; and those who acquired both) use the same voting strategy, denoted $w_r(r) : \{0,1\} \to [0,1]$, $w_s(s_i) : \{0,1\} \to [0,1]$, $w_0 \in [0,1]$, and $w_{rs}(r, s_i) : \{0,1\} \times \{0,1\} \to [0,1]$, where $w_r(\cdot)$, $w_s(\cdot)$, w_0, and $w_{rs}(\cdot)$ denote the probability of voting "for" given the respective information set. Second, since the model is fully symmetric in states and signals, we look for equilibria that are symmetric around the state: $w_s(s_i) = 1 - w_s(1 - s_i)$, $w_r(r) = 1 - w_r(1 - r)$, $w_0 = \frac{1}{2}$, and $w_{rs}(r, s_i) = 1 - w_{rs}(1 - r, 1 - s_i)$ $\forall s_i \in \{0,1\}$ and $\forall r \in \{0,1\}$.[13] In what follows, we refer to symmetric equilibria as simply equilibria.[14]

We assume that shareholders cannot abstain from voting on the proposal. This assumption matches reality: in practice, institutional investors rarely abstain from voting, probably because of the fear of violating their fiduciary duties or of being perceived as uninformed. For example, according to our calculations based on the ISS Voting Analytics database for 2003-2012, mutual funds abstain in less than 1% of cases.[15]

The model described in this section is stylized. The benefit is that it leads to tractable solutions and clearly shows the underlying economic forces: the valuable social function of a proxy advisor in providing investors with a new information acquisition technology and the inefficiencies in the choice of information acquisition technologies due to a collective action problem. The cost of tractability is that the model does not incorporate several features of the proxy advisory industry. In Section 7, we discuss how our model can be extended to account for some of these features.

[13] The symmetry assumption allows us to eliminate "uninformative" equilibria, where all shareholders remain uninformed and then all vote in the same direction.

[14] In particular, when we say there is a unique equilibrium, we mean a unique symmetric equilibrium.

[15] Moreover, the equilibrium of our model will also be an equilibrium if we extend the model by allowing each shareholder to abstain from voting and assume that in the event of a tie, the proposal is implemented randomly. Consider an uninformed shareholder and note that his vote only matters if the votes of other shareholders are split equally. Conditional on this event, both states are equally likely and hence the shareholder is indifferent between it being accepted or rejected. If the shareholder abstains from voting, the proposal is implemented randomly, uncorrelated with the state; if the shareholder does not abstain from voting, he randomizes between voting for and against and hence the implementation of the proposal is also independent of the state. Hence, the uninformed shareholder is indifferent between abstaining and not abstaining, and thus our equilibrium indeed continues to exist in this extended model.

2.1 Benchmark: Voting without the proxy advisory firm

As a benchmark, it is useful to consider shareholder voting in the absence of the advisor. In this case, the model is an extension of the standard problem of strategic voting,[16] augmented by the information acquisition stage. A variation of this problem has been studied by Persico (2004).

An equilibrium is given by probability $q \in [0,1]$ with which each shareholder acquires a private signal; function $w_s(s)$, the probability of voting "for" given signal s; and probability $w_0 = \frac{1}{2}$ of voting "for" given no information.

In equilibrium, each shareholder who acquires a private signal votes according to his signal. Indeed, if the shareholder always votes in the same way regardless of his signal, he is better off not paying for the signal in the first place. Similarly, if the shareholder mixes (and hence is indifferent) between voting according to his signal and against it for at least one realization of the signal, then his utility would not change if he voted in the same way regardless of his signal, so he is again better off not acquiring the signal.

Given the equilibrium at the voting stage, we can solve for the equilibrium at the information acquisition stage. Consider shareholder i contemplating whether to acquire a private signal, given that he expects each other shareholder to acquire a private signal with probability q. Suppose that the shareholder's private signal is $s_i = 1$. Whether he is informed or not only makes a difference if his vote is pivotal, i.e., the number of "for" votes among other shareholders is exactly $\frac{N-1}{2}$.[17] Let us denote this set of events by PIV_i. In this case, by acquiring the signal, the shareholder votes "for" for sure, instead of randomizing between voting "for" and "against," so his utility from being informed is $\frac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right]$. Similarly, conditional on his private signal being $s_i = 0$, the shareholder's utility from being informed is $-\frac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]$. Overall, the shareholder's value of acquiring a signal is

$$\begin{aligned} V(q) = &\Pr(s_i = 1)\Pr(PIV_i|s_i = 1)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] \\ &- \Pr(s_i = 0)\Pr(PIV_i|s_i = 0)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]. \end{aligned} \tag{4}$$

It is useful to define function $P(x,n,k)$ as the probability that the proposal gets k votes

[16] Maug and Rydqvist (2009) provide evidence consistent with shareholders voting strategically.

[17] In practice, the probability of a "close vote" is non-negligible. For example, Fos and Jiang (2015) document that in proxy contests, the median difference between the number of shares cast in favor of the winning party and the number of shares for the losing party, normalized by the number of shares outstanding, is 24%, and in 10% of proxy contests, a reallocation of 2% of voting rights from winners to losers could flip the voting outcome.

out of n when each shareholder independently votes for the proposal with probability x:

$$P(x, n, k) \equiv C_n^k x^k (1-x)^{n-k}, \tag{5}$$

where $C_n^k = \frac{n!}{k!(n-k)!}$ is the binomial coefficient. Using the symmetry of the setup and Bayes' rule, we can write $V(q)$ as (see the proof of Proposition 1 for the derivation):

$$V(q) = (p - \frac{1}{2})P(qp + (1-q)\frac{1}{2}, N-1, \frac{N-1}{2}) = (p - \frac{1}{2})C_{N-1}^{\frac{N-1}{2}} \left(\frac{1}{4} - q^2(p - \frac{1}{2})^2\right)^{\frac{N-1}{2}}. \tag{6}$$

The intuition behind (6) is simple. Consider one shareholder. Any other shareholder acquires his private signal with probability q and hence votes correctly with probability $qp + (1-q)\frac{1}{2}$: the probability of a correct vote equals the precision of the signal p if the shareholder gets informed, and equals $\frac{1}{2}$ if he does not. Thus, the votes of other $N-1$ shareholders are split with probability $P\left(qp + (1-q)\frac{1}{2}, N-1, \frac{N-1}{2}\right)$. Conditional on this event, the value of the signal to the shareholder equals $p - \frac{1}{2}$, implying that the expected value from getting informed is (6). The value of information $V(q)$ is decreasing in the number of shareholders N or, equivalently, increasing in the stake of each shareholder. This is because with more shareholders, the shareholder's vote is less likely to determine the decision, reducing his incentives to acquire information. In addition, $V(q)$ is decreasing in the probability q with which other shareholders acquire their private signals. Intuitively, as other shareholders become more informed, they are more likely to all vote in the same way, which reduces the chances of a close vote when the shareholder's information becomes critical.

In deciding whether to acquire the private signal, shareholder i compares the expected value from the signal, $V(q)$, with cost c and acquires the signal if and only if $V(q) \geq c$. Since $V(q)$ is strictly decreasing in q, the equilibrium probability q is determined as a unique solution to $V(q) = c$, unless c is very low or very high. If c is very low or very high, then either all shareholders acquire information or none of them do. This equilibrium is summarized in Proposition 1 below.

Proposition 1 (equilibrium without the advisor). *There exists a unique equilibrium.*

Each shareholder acquires a private signal with probability q^*, given by

$$q^* = \begin{cases} 1, & \text{if } c \leq \underline{c} \equiv V(1) = \left(p - \frac{1}{2}\right) C_{N-1}^{\frac{N-1}{2}} \left(\frac{1}{4} - \left(p - \frac{1}{2}\right)^2\right)^{\frac{N-1}{2}}, \\ q_0^* \equiv \frac{2}{2p-1}\Lambda, & \text{if } c \in (\underline{c}, \bar{c}), \\ 0, & \text{if } c \geq \bar{c} \equiv V(0) = \left(p - \frac{1}{2}\right) C_{N-1}^{\frac{N-1}{2}} 2^{1-N}. \end{cases} \tag{7}$$

where $\Lambda \equiv \sqrt{\frac{1}{4} - \left(\frac{c}{p-\frac{1}{2}} \frac{1}{C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}$. *At the voting stage, a shareholder with signal s_i votes "for" ($v_i = 1$) if $s_i = 1$ and "against" ($v_i = 0$) if $s_i = 0$, and an uninformed shareholder votes "for" with probability* 0.5.

In what follows, we assume that $c \in (\underline{c}, \bar{c})$, that is, the interior solution occurs in the model without the advisor.

Assumption 1. *$c \in (\underline{c}, \bar{c})$, so that $q^* \in (0, 1)$ in the model without the advisor.*

The rationale for Assumption 1 is simple: we want to focus on the cases where private information acquisition is a relevant margin. If $c > \bar{c}$, the problem becomes trivial: private information acquisition never occurs. In this case, the advisor always creates value, since there is no crowding out of private information and a partially informed vote is strictly better than a completely uninformed one. Note, however, that given the 2003 SEC rule, an institutional investor that does not acquire any information and votes uninformatively potentially exposes itself to legal risk for violating its fiduciary duty of voting in the best interests of its clients. Given that, it is plausible to assume that even in the absence of a proxy advisor, some private information acquisition would occur. Similarly, the case $c < \underline{c}$ is not empirically plausible because in practice many shareholders voted uninformatively prior to the emergence of proxy advisory firms. Note that both $\underline{c}$ and $\bar{c}$ depend negatively on N and approach zero in the limit as $N \to \infty$, i.e., as ownership becomes infinitely dispersed (see the Online Appendix for the proof). Thus, assumption $c < \bar{c}$ imposes a restriction that ownership is not too dispersed.[18] In this sense, another interpretation of Assumption 1 is that we focus on information acquisition decisions of shareholders who are not too large and not too small.

[18] In Section 4.5, we consider what happens under very dispersed ownership, i.e., when $q^* = 0$.

To measure the quality of decision-making, we use the equilibrium expected per-share value of the proposal (in what follows, we refer to it simply as firm value). The proof of Proposition 1 shows that firm value in the absence of the advisor is given by

$$V_0 = \sum_{k=\frac{N+1}{2}}^{N} P(q_0^* p + \frac{1-q_0^*}{2}, N, k) - \frac{1}{2} = \sum_{k=\frac{N+1}{2}}^{N} P(\frac{1}{2} + \Lambda, N, k) - \frac{1}{2}. \tag{8}$$

3 Voting with the proxy advisory firm

In this section, we introduce the advisor and solve for the equilibria of the game, taking as given fee $f > 0$ set by the advisor (we analyze the fee that maximizes the advisor's profits in the next section). We solve the model by backward induction. First, we find the equilibria at the voting stage. Next, we solve for the equilibrium information acquisition decisions of the shareholders.

3.1 Voting stage

Following the same argument as in Section 2.1, if a shareholder acquires exactly one signal (private or advisor's), he follows it with probability one. Otherwise, the value of this signal to the shareholder would be zero and he would be better off not paying for it in the first place.

In addition, it cannot occur in equilibrium that a shareholder acquires both his private signal and the proxy advisor's signal. Intuitively, when the signals disagree, the shareholder follows the more informative (conditional on the event that his vote matters) signal, so he would be better off not buying the less informative signal. Indeed, suppose, for example, that such a shareholder votes "for" when $r = 1$ and $s_i = 0$. By symmetry of the equilibrium, if the situation is reversed, i.e., $r = 0$ and $s_i = 1$, the shareholder votes "against." This, however, implies that the shareholder ignores his private signal and hence would be strictly better off if he only acquired the proxy advisor's signal. The proof of Proposition 2 presents this argument formally. While the fact that a shareholder does not acquire both signals is a convenient feature of the model that makes the analysis tractable, the intuition behind many effects does not depend on it. We discuss this property in more detail in Section 7.

Therefore, for information acquisition decisions to be consistent with equilibrium, the equilibrium at the voting stage must take the following form: A shareholder who acquired a private signal votes according to it, a shareholder who acquired the advisor's recommendation votes according to it, and a shareholder who stayed uninformed randomizes between voting "for" and "against" with equal probabilities:

Proposition 2 (voting with the advisor). *In equilibrium, shareholders' strategies at the voting stage must be* $w_s(s_i) = s_i$, $w_r(r) = r$, *and* $w_0 = \frac{1}{2}$.

Let q_s and q_r denote probabilities with which each shareholder buys a private signal and the proxy advisor's signal, respectively. Then, the probability that a shareholder stays uninformed is $1 - q_s - q_r$.

3.2 Information acquisition stage

Having solved for the equilibrium at the voting stage, we calculate the value of information to a shareholder for given q_r and q_s. Using the same arguments as in Section 2.1, we show in the Online Appendix that the values to any shareholder from acquiring a private signal and the recommendation of the advisor are, respectively, given by

$$V_s(q_r, q_s) = (p - \frac{1}{2})(\pi\Omega_1(q_r, q_s) + (1-\pi)\Omega_2(q_r, q_s)) \tag{9}$$

$$V_r(q_r, q_s) = \frac{1}{2}(\pi\Omega_1(q_r, q_s) - (1-\pi)\Omega_2(q_r, q_s)), \tag{10}$$

where $\Omega_1(q_r, q_s) \equiv P(\frac{1+q_r}{2} + q_s(p - \frac{1}{2}), N-1, \frac{N-1}{2})$ and $\Omega_2(q_r, q_s) \equiv P(\frac{1-q_r}{2} + q_s(p - \frac{1}{2}), N-1, \frac{N-1}{2})$ denote the probabilities that the shareholder is pivotal when the advisor's recommendation is correct ($r = \theta$) and when it is incorrect ($r \neq \theta$), respectively. The intuition again follows from the fact that whether a shareholder is informed or not makes a difference only if the shareholder's vote is pivotal for the outcome. First, consider (9). Since all other signals are conditionally independent of the shareholder's private signal, the value of the signal to the shareholder equals the probability that the shareholder is pivotal (the term in the second brackets) times the value of the signal in this case $(p - \frac{1}{2})$. Second, consider (10). Now, as long as $q_r > 0$, the acquired signal is no longer conditionally independent of other shareholders' votes because some other shareholders acquire the advisor's recommendation as well. When the advisor is correct (incorrect), the value to the shareholder from buying and following the advisor's recommendation conditional on being pivotal is $\frac{1}{2}$ $(-\frac{1}{2})$ because the shareholder makes the correct (incorrect) decision instead of randomizing between them with probability $\frac{1}{2}$. Combining these two cases gives (10).

When deciding which signal to acquire, if any, a shareholder compares $V_s(q_r, q_s) - c$ with $V_r(q_r, q_s) - f$ and with zero, and chooses the option with the highest payoff. The fact that a shareholder's information is only valuable to him when he is pivotal leads to an interesting interdependence in information acquisition decisions of different shareholders. To see it, consider the relative value of the two signals to a shareholder. Dividing (10) by (9) and rearranging the terms,

$$\frac{V_r(q_r, q_s)}{V_s(q_r, q_s)} = \frac{\pi \frac{\Omega_1(q_r, q_s)}{\pi\Omega_1(q_r, q_s) + (1-\pi)\Omega_2(q_r, q_s)} - \frac{1}{2}}{p - \frac{1}{2}}. \tag{11}$$

The right-hand side of (11) reflects the ratio of the precisions of the two signals, π and p, adjusted by what the shareholder learns about the precision of each signal from the fact that the votes of others are split equally. If some shareholders follow the advisor ($q_r > 0$), the fact that the vote is split implies that among shareholders who do not follow the advisor, more vote against the advisor's recommendation than with it. This fact does not reveal any information about whether the advisor's recommendation is correct if no shareholder acquires a private signal: $\Omega_1(q_r, 0) = \Omega_2(q_r, 0)$. However, if some shareholders acquire private signals ($q_s > 0$), a split vote is a signal that the advisor's recommendation is more likely to be incorrect ($r \neq \theta$), since a split vote is more likely when private signals of shareholders disagree with the advisor's recommendation than when they agree with it: $\Omega_2(q_r, q_s) \geq \Omega_1(q_r, q_s)$. Therefore, as long as $q_r > 0$ and $q_s > 0$, the information content from being pivotal lowers the shareholder's assessment of the precision of the advisor's recommendation, which is represented by the second multiple in the numerator of (11). Note also that the event of being pivotal does not provide any additional information about the precision of the shareholder's private signal, and hence the denominator of (11) just includes the unadjusted precision p.

This learning from the event of being pivotal leads to complementarity in shareholders' information acquisition decisions in the following sense. Suppose, for simplicity, that the two signals have the same cost ($f = c$). First, suppose that a shareholder expects all other shareholders to either follow the advisor or vote uninformatively, i.e., $q_s = 0$. Then, according to the logic above, the fact that the shareholder is pivotal conveys no new information about whether the advisor's recommendation is correct or not. As a consequence, if the advisor's recommendation is even a tiny bit more precise than the private signal ($\pi > p$), the shareholder prefers to buy the recommendation from the advisor over buying a private signal for any q_r, i.e., even if a lot of other shareholders are also expected to follow the advisor. In contrast, if a shareholder expects some other shareholders to acquire private signals, i.e., $q_s > 0$, then he infers that conditional on the votes of other shareholders being split, the advisor's recommendation is correct with probability less than π. This, all else equal, pushes him in the direction of buying a private signal over the proxy advisor's recommendation. Thus, other shareholders' decisions to acquire private signals induces the shareholder to acquire a private signal as well. As we show below, this complementarity leads to a multiplicity of equilibria. In particular, in the special case where the advisor's signal has both the same cost and the same precision as shareholders' private signals ($f = c$ and $\pi = p$), we have an extreme form of complementarity: all shareholders who become informed acquire the same type of information. In particular, as Lemma 1 below shows, there exist two equilibria: in the first, all shareholders who become informed acquire private signals ($q_r = 0$), and in the second, all shareholders who become informed acquire the advisor's signal ($q_s = 0$).

Given (9) and (10), we can determine the equilibrium information acquisition strategies. If $q_r = 0$, the problem is identical to the benchmark model of Section 2.1, so $q_s = q_0^*$. For this to be an equilibrium, it must be that $V_r\left(0, q_0^*\right) \leq f$. If $q_r > 0$, i.e., some shareholders acquire the advisor's recommendation, the following two cases are possible:

- **Case 1: Incomplete crowding out of private information acquisition ($q_s > 0$).** Shareholders randomize between acquiring the advisor's recommendation, the private signal, and staying uninformed: $q_r > 0$, $q_s > 0$, and $q_s + q_r \leq 1$.[19] In this case, q_r and q_s are found from

$$V_s\left(q_r, q_s\right) - c = V_r\left(q_r, q_s\right) - f \geq 0, \tag{12}$$

with equality if $q_s + q_r < 1$.

- **Case 2: Complete crowding out of private information acquisition ($q_s = 0$).** Shareholders randomize between acquiring the advisor's recommendation and staying

[19] More specifically, if $q_s + q_r < 1$, shareholders randomize between acquiring the advisor's recommendation, acquiring the private signal, and staying uninformed, and if $q_s + q_r = 1$, all shareholders become informed and randomize between acquiring the advisor's recommendation and the private signal.

uninformed. Probability q_r is given by $V_r(q_r, 0) = f$, which implies

$$q_r = \sqrt{1 - 4\left(\frac{f}{C_{N-1}^{\frac{N-1}{2}}(\pi - \frac{1}{2})}\right)^{\frac{2}{N-1}}}. \tag{13}$$

For this to be an equilibrium, it must be that $V_s(q_r, 0) \leq c$.

The next lemma describes the equilibria for all values of f.

Lemma 1. *For a given fee $f > 0$, the set of equilibria is as follows:*

1. If $f > \bar{f} \equiv \frac{2\pi-1}{2p-1}c$, there is a unique equilibrium, which is identical to that in the benchmark model: $q_s = q_0^*$ and $q_r = 0$.

2. If $f \in [\underline{f}, \bar{f})$, where $\underline{f}$ is defined in the Appendix, there co-exist two types of equilibria: (1) equilibrium with incomplete crowding out of private information acquisition $(q_r, q_s) > 0$ and (2) equilibrium with complete crowding out of private information acquisition: $q_s = 0$, $q_r \in (0, 1)$. For $f = \bar{f}$, these equilibria converge, so that there co-exist two equilibria: one where only private information is acquired: $q_s = q_0^*$, $q_r = 0$, and one with complete crowding out: $q_s = 0$, $q_r \in (0, 1)$.

3. If $f < \underline{f}$, the unique equilibrium features complete crowding out of private information acquisition: $q_s = 0$, $q_r \in (0, 1)$.

The structure of the equilibrium is intuitive. If fee f is so high ($f \geq \bar{f}$) that the cost-to-precision ratio of the advisor's recommendation ($\frac{f}{\pi-0.5}$) exceeds that of the private signal ($\frac{c}{p-0.5}$), no shareholder finds it optimal to acquire its recommendation. If the advisor's fee is very low, $f < \underline{f}$, no shareholder finds it optimal to use the private information acquisition technology. Finally, in the intermediate range of f, there exist equilibria in which both types of signals are acquired in equilibrium. In this region, there are multiple equilibria for the reason described above.

3.3 Quality of decision-making for a given fee

Given the equilibrium at the information acquisition and voting stages, we can compute the per-share expected value of the proposal (firm value), which measures the quality of decision-making with the advisor. Comparing it with value (8) in the benchmark case allows us to examine whether the presence of the advisor increases firm value for a given fee f. The following proposition shows the effect of the proxy advisor on the informativeness of decision-making for any fixed fee f:

Proposition 3 (quality of decision-making for a given fee). *Fix fee f.*

1. *In any equilibrium with incomplete crowding out of private information acquisition, firm value is strictly lower than in the benchmark case.*

2. *Consider equilibrium with complete crowding out of private information acquisition. There exists a threshold $\pi^*(f) > \frac{1}{2} + \frac{f}{c}\left(p - \frac{1}{2}\right)$ such that firm value is lower than in the benchmark case if and only if $\pi \leq \pi^*(f)$.*

Proposition 3 shows that the presence of the advisor harms the quality of decision-making unless there is complete crowding out of private information acquisition and the advisor's signal is sufficiently precise. In both cases, this happens because information acquisitions decisions that are privately optimal from each shareholder's perspective are not optimal from the perspective of firm value maximization, leading to inefficient crowding out of private information acquisition and suboptimal voting decisions.

To see this intuition in the simplest way, consider first the second part of the proposition, i.e., the case of complete crowding out, and suppose that $f = c$. A shareholder who decides which signal to acquire, conditions his decision on the event that the votes of other shareholders are split. As discussed in Section 3.2, because no other shareholder acquires private information, the fact that the votes are split does not add anything to the shareholder's prior beliefs about the informativeness of the advisor's signal. Hence, given that the two signals are equally costly, the shareholder simply compares their precisions π and p. In particular, he finds it privately optimal to acquire the advisor's signal as long as it is more precise, $\pi > p$, even if many other shareholders follow the advisor as well. This, however, is inefficient if the advisor's signal is only marginally more precise than the private signal. Indeed, if many shareholders are following the advisor, they all vote in the same way, and their mistakes are perfectly correlated. In contrast, when shareholders are following their private signals, their mistakes are independent conditional on the state, and hence the voting outcome is more likely to be efficient.

In equilibrium with incomplete crowding out, some shareholders acquire private signals, but their fraction is small enough, so that informativeness of voting goes down. To see the intuition, note that a shareholder's private value of acquiring his own signal depends on the precision of the signal and the probability that the votes of other shareholders will be split. This probability is lower if there is a high correlation between the votes of other shareholders. This correlation can arise for two reasons: either because shareholders have information about the fundamentals and thus make correlated informative decisions, or because shareholders rely on the same noisy signal and thus make correlated mistakes. While the source of correlation does not matter for the shareholder's private value of his signal, it is important for firm value: correlation in votes due to each vote being correlated with the true state θ is more efficient than correlation in votes due to many votes reflecting the same error term (which is the case when many shareholders follow the advisor). To see this formally, consider the case $q_r + q_s < 1$. Then, the equilibrium probability that a shareholder is pivotal is determined by condition $\Pr(PIV_i)\left(p - \frac{1}{2}\right) = c$: because a shareholder must be indifferent between acquiring a private signal and staying uninformed on the margin, the probability that she is pivotal is the same both with and without the advisor. However, in the benchmark case without the advisor, $\Pr(PIV_i)$ is affected by the correlation in votes due to the correlation in shareholders' private signals. In contrast, with the advisor, part of the correlation arises due to correlated mistakes from the reliance on the advisor's recommendation. Thus, inefficient correlation in votes due to correlated mistakes crowds out efficient correlation in votes due to reliance on fundamentals, leading to lower firm value.

Importantly, the result that the presence of the advisor can be detrimental for firm value crucially depends on the coordination problem due to collective decision-making by shareholders. If the firm had only one shareholder or if shareholders could coordinate their information acquisition and voting decisions, the presence of an additional valuable signal from the advisor would always be beneficial.

4 Pricing of information by the proxy advisor

4.1 Equilibrium selection

As Proposition 3 shows, the result that the presence of the advisor can improve the informativeness of decision-making only if its recommendation is sufficiently precise holds regardless of the equilibrium selection. However, to have a well-defined problem of pricing of information, we need to take a stand on which equilibrium is played in the range of fees where multiple equilibria exist. As the next lemma shows, the equilibria can be ranked in their shareholder welfare, provided that we marginally strengthen Assumption 1:

Lemma 2. *Suppose that $c \in (\hat{c}, \bar{c})$, where $\hat{c}$ is defined in the Appendix. Then, in the range $f \in [\underline{f}, \bar{f})$, all equilibria can be ranked in their shareholder welfare (expected value of the proposal minus information acquisition costs). Specifically, there exist three equilibria, with equilibrium (a) having the highest and (c) having the lowest shareholder welfare: (a) equilibrium with incomplete crowding out of private information acquisition and $q_r \leq (2p-1) q_s$, given by (23) in the Appendix; (b) equilibrium with incomplete crowding out of private information acquisition and $q_r \geq (2p-1) q_s$, given by (24) in the Appendix; (c) equilibrium with complete crowding out of private information acquisition: $q_s = 0$ and q_r given by (13). Equilibria (a) and (b) coincide when $f = \underline{f}$.*

Restriction on cost c in Lemma 2 is very similar to Assumption 1. In particular, condition $c > \underline{c}$ in Assumption 1 ensures that some shareholders stay uninformed with positive probability in the benchmark case without the advisor. Similarly, condition $c > \hat{c}$ in Lemma 2 ensures that some shareholders stay uninformed with positive probability in the model with the advisor: $q_r + q_s < 1$.[20]

Given Lemma 2, it is natural to assume that when the advisor's fee is in the intermediate range, $f \in [\underline{f}, \bar{f})$, so that multiple equilibria exist at the information acquisition stage, shareholders coordinate on the equilibrium in which shareholder welfare is maximized. Since shareholders are identical, this selection is equivalent to the Pareto-dominance criterion, according to which an equilibrium is not selected if there exists another equilibrium with higher payoffs for all players in the subgame. Thus, we impose the following assumption for the remainder of the paper:

Assumption 2 (equilibrium selection). *$c \in (\hat{c}, \bar{c})$ and, when $f \in [\underline{f}, \bar{f})$, shareholders coordinate on the equilibrium that maximizes shareholder welfare.*

Assumption 2 and Lemmas 1 and 2 imply the following equilibrium in the information

[20] By definition of $\hat{c}$, $\hat{c} \geq \underline{c}$, but for many parameter values, $\hat{c} = \underline{c}$.

acquisition subgame:

Proposition 4 (equilibrium information acquisition). *For a given fee f, the equilibrium at the information acquisition stage is as follows:*

1. *If $f \geq \bar{f}$, then $q_r = 0$ and $q_s = q_0^* \in (0, 1)$, given by (7).*
2. *If $f \in [\underline{f}, \bar{f})$, then $q_r \in (0, (2p-1)\, q_s]$ and $q_s \in (0, 1 - q_r)$, which satisfy (12) with strict equality and are given by (23) in the Appendix.*
3. *If $f < \underline{f}$, then $q_s = 0$ and $q_r \in (0, 1)$, given by (13).*

The equilibrium firm value is decreasing in f if $f < \underline{f}$, increasing in f if $f \in [\underline{f}, \bar{f})$, and is constant in f if $f \geq \bar{f}$.

Figure 2 illustrates the proposition. In this example, there are 35 shareholders, the private information acquisition cost is 1.5% of the potential value of the proposal per shareholder, and the precisions of the private signal and the advisor's recommendation are $p = 0.65$ and $\pi = 0.75$, respectively. When the advisor's fee exceeds $\bar{f} = 2.5\%$, the precision-to-price ratio of the advisor's signal is below that of the private signal. In this case, no shareholder acquires information from the advisor, and the equilibrium is identical to the benchmark case. In particular, a shareholder acquires a private signal with probability 44.5% and remains uninformed with probability 55.5%. When the advisor's fee is between $\underline{f} \approx 1.6\%$ and $\bar{f} = 2.5\%$, incomplete crowding out of private information acquisition occurs in equilibrium. In this range, as fee f decreases further, the probability that a shareholder acquires the advisor's recommendation (private signal) increases (decreases), and the probability that a shareholder remains uninformed increases. Hence, a lower fee leads to additional crowding out of private information production, which harms firm value: the right panel of the figure shows that firm value is increasing in f in this range. Finally, when the fee charged by the advisor is below $\underline{f} \approx 1.6\%$, the advisor completely crowds out private information acquisition. A lower fee in this range leads to more shareholders buying the advisor's recommendation and has no effect on private information production, since it is already absent. Hence, as the right panel of Figure 2 demonstrates, firm value is decreasing in f in this range.[21]

[21] In the example of Figure 2, firm value at $f \geq \overline{f}$ exceeds firm value at $f \to 0$, but this is specific to the parameters in this example.

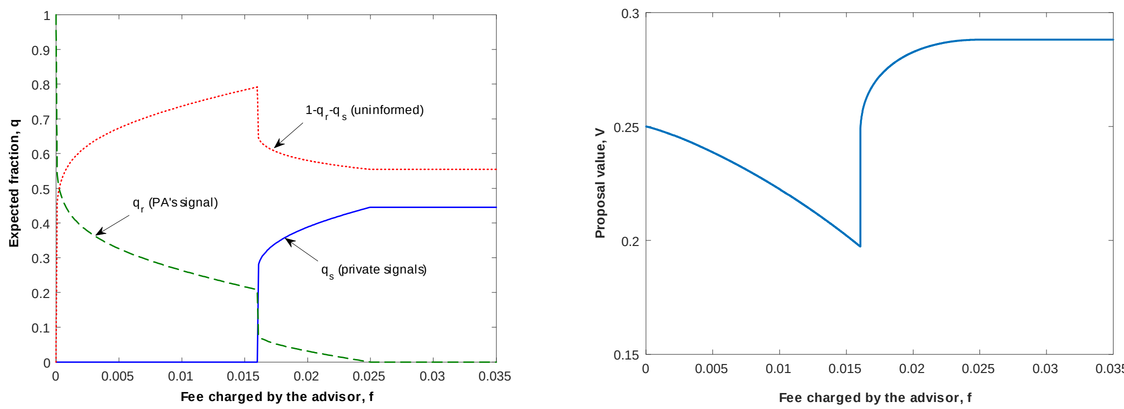

Figure 2. Equilibrium information acquisition. The left figure plots the equilibrium information information acquisition as a function of the fee f charged by the advisor. The solid line depicts probability q_s that a shareholder acquires a private signal. The dashed line depicts probability q_r that a shareholder acquires the advisor's recommendation. The dotted line depicts the probability that a shareholder remains uninformed. The right figure plots the corresponding expected value of the proposal. The parameters are $N = 35$, $p = 0.65$, $\pi = 0.75$, and $c = 0.015$.

4.2 Equilibrium price of information

In this section, we study strategic fee setting by the monopolistic advisor. The advisor maximizes its profits, taking into account how its fee affects shareholders' information acquisition decisions. Proposition 4 implies that the demand function for the advisor's recommendation is given by

$$q_r(f) = \begin{cases} q_r^H(f), & \text{if } f < \underline{f}, \\ q_r^L(f), & \text{if } f \in [\underline{f}, \bar{f}), \\ 0, & \text{if } f \geq \bar{f}, \end{cases} \tag{14}$$

where $q_r^H(f)$ corresponds to complete crowding out of private information and is given by (13), and $q_r^L(f) < q_r^H(f)$ corresponds to incomplete crowding out of private information and is given by (23) in the Appendix. An example of this demand function is shown in Figure 2. The optimal fee chosen by the advisor, denoted f^*, maximizes its expected revenues $fq_r(f)$. We make a technical assumption that the space of feasible prices is discrete with infinitesimal increments, e.g., the advisor can set the fee in the increments of a penny.[22]

[22] Without this restriction the optimal fee may not exist, because the advisor may prefer to set the fee as close to $\underline{f}$ as possible, but not $\underline{f}$. This restriction is often imposed in models of Bertrand competition to

Consider the unconstrained problem of the advisor, $f = \arg\max fq_r^H(f)$, i.e, the problem where the advisor faces no competition from the private information acquisition technology. The proof of Proposition 5 shows that the function $fq_r^H(f)$ is inverse U-shaped in f and has a maximum at

$$f_m \equiv (\pi - \frac{1}{2})P(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N-1, \frac{N-1}{2}), \tag{15}$$

which corresponds to $q_r = \frac{1}{\sqrt{N}}$.

It follows that depending on the parameters, one of the following three cases is possible. If $f_m < \underline{f}$, which happens when the precision of the advisor's signal is sufficiently high and the private information acquisition technology is sufficiently costly, then the advisor sets $f^* = f_m$. If $f_m \geq \underline{f}$, then one of the two scenarios is possible. First, the advisor could select the maximum possible fee given which there is complete crowding out of private information acquisition - the price that is just an infinitesimal increment above $\underline{f}$. This strategy is akin to "limit pricing" in industrial organization, where the incumbent sets its price just low enough to make it unprofitable for a potential entrant to enter the market. Second, the advisor could select fee $f^* > \underline{f}$ that maximizes its revenues conditional on incomplete crowding out of private information acquisition.

4.3 Equilibrium firm value

Denote $V^*(\pi)$ the expected value of the proposal given the equilibrium fee f^* chosen by the advisor. Under what conditions is $V^*(\pi)$ higher than in the benchmark model without the advisor? The proof of Proposition 5 shows that it can happen only if the advisor chooses fee f^* that maximizes its unconstrained problem. In other words, firm value can only be higher than in the benchmark case if the advisor sets fee $f^* = f_m$, and each shareholder acquires the advisor's signal with probability $\frac{1}{\sqrt{N}}$ and remains uninformed otherwise. The proof of Proposition 5 shows that the expected value of the proposal in this case is given by

$$V^*(\pi) = (\pi - \frac{1}{2})[2\sum_{k=\frac{N+1}{2}}^{N} P(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k) - 1]. \tag{16}$$

To compare it with firm value in the benchmark case, which is given by V_0 in (8), define $\pi^{**} \equiv \sum_{k=\frac{N+1}{2}}^{N} P(p_0, N, k)$, where $p_0 \equiv pq_0^* + \frac{1-q_0^*}{2}$ and q_0^* is the benchmark equilibrium

ensure existence of reaction functions, e.g., in Maskin and Tirole (1988).

probability of a shareholder acquiring private information, given by (7). Intuitively, π^{**} is the equilibrium probability of making a correct decision in the benchmark model without the advisor. Then $V_0 = \pi^{**} - \frac{1}{2}$, and hence condition $V^*(\pi) > V_0$ holds if and only if

$$\pi > \tilde{\pi} \equiv \frac{1}{2} + \frac{\pi^{**} - \frac{1}{2}}{2\sum_{k=\frac{N+1}{2}}^{N} P(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k) - 1}. \tag{17}$$

Interestingly, since the denominator in the second term of (17) is below one, $\tilde{\pi}$ exceeds one if π^{**} is sufficiently high, that is, if private signals are relatively cheap and a sufficient fraction of shareholders acquire information in the benchmark case. In this case, the advisor always harms firm value, even if $\pi = 1$, i.e., its information is perfectly precise. Intuitively, even if its recommendation is extremely precise, the advisor never finds it optimal to sell it to all shareholders: its profits are higher if it sells the recommendation to fewer shareholders but charges a higher fee. As a consequence, many shareholders remain uninformed and hence the advisor's information does not get perfectly incorporated in the vote. If the quality of decision-making without the advisor is sufficiently high, this effect implies that the presence of the advisor harms firm value even if the advisor is perfectly informed. These results are summarized in the following proposition.

Proposition 5 (equilibrium quality of decision-making). *Firm value in the presence of the advisor is strictly lower than in the benchmark case if and only if the precision of the advisor's signal π is below $\tilde{\pi}$ given by (17). In particular, if $(2p-1)\, q_0^* > \frac{1}{\sqrt{N}}$, firm value is strictly lower than in the benchmark case for any precision $\pi \in (\frac{1}{2}, 1]$ of the advisor's signal.*

Figure 3 illustrates how the equilibrium fee charged by the advisor and the expected firm value relative to the benchmark case depend on the precision of the advisor's recommendation. Figures 3a-3c use the same parameters as Figure 2: there are 35 shareholders, the private information acquisition cost is 1.5% of the potential value of the proposal per share, and the precision of the private signal is $p = 0.65$. When the advisor's information is sufficiently precise, $\pi > 0.84$, it can set the fee in a way as if it faced no competition from the private information acquisition technology: $f^* = f_m$, the unconstrained optimal fee. When the advisor's information becomes less precise, $\pi < 0.84$, shareholders would acquire private information, had the advisor set the fee at f_m. To prevent this, the advisor engages in limit pricing by setting the fee at the highest possible level that allows it to crowd out private information acquisition. As a result of this pricing strategy, shareholders do not acquire private information for any $\pi > 0.64$. Finally, when the precision of the advisor's recommendation falls below 0.64, both types of signals are acquired in equilibrium. Figure 3c illustrates the first statement of Proposition 5 and shows that the expected value of the proposal is higher than in the benchmark case only if there is complete crowding out of private information acquisition and the advisor's signal is sufficiently precise, $\pi > 0.92$.

Finally, Figure 3d illustrates the role of strategic pricing by the proxy advisor. It compares firm value in the model with endogenous pricing to firm value in the model where the advisor's fee f is fixed at some exogenous level (for illustrative purposes, we pick $f = \underline{f}$ at the lowest π at which the equilibrium features limit pricing). When the equilibrium features incomplete crowding out of private information acquisition (the left part of the figure), strategic pricing by the advisor leads to higher firm value than in the case with an exogenous fee – this is manifested by the solid line being above the dotted line in this region. Intuitively, as π increases, the advisor optimally charges a higher fee, and hence there is less crowding out of private information production than if the fee were fixed. In contrast, when the equilibrium features complete crowding out of private information acquisition (the right part of the figure), firm value under endogenous pricing is lower than under exogenous pricing. Intuitively, as π increases, the advisor adjusts its fee upwards, and hence fewer shareholders become informed than if the fee stayed constant. This effect also leads to the second statement of Proposition 5 illustrated in this figure: when shareholders' private signals are sufficiently cheap (we pick $c = 0.0075$ for this figure), the presence of the advisor hurts firm value even if its information is perfectly precise (when $\pi = 1$, the solid line is below the dashed line).

It is also interesting to compare the equilibrium in our model to the equilibrium when shareholders have no option to buy private signals. In this extreme case, the advisor would always charge the unconstrained optimal fee f_m, given by (15). Hence, if π is sufficiently high (the right region in Figure 3b), the equilibrium is the same whether or not shareholders have an option to buy private signals. If π is lower, comparison of the two equilibria shows that the ability to buy private signals unambiguously increases informativeness of decision-making. In particular, if there is accommodation of private information acquisition (the left region in Figure 3b), shareholders' ability to buy private signals improves decision-making directly by incorporating private signals into the vote. If there is limit pricing (the middle region in Figure 3b), it improves decision-making indirectly by forcing the advisor to lower the price of its information, which induces more shareholders to buy it. Thus, unlike the presence of the advisor, the presence of the option to buy private signals always weakly improves the quality of decision-making.

4.4 Sources of inefficiency

As the previous section demonstrates, there are two sources of inefficiencies in our setting: inefficient information acquisition due to the collective action problem and strategic pricing by the monopolistic advisor. To illustrate these sources of inefficiencies better, we compare the equilibrium of the model to the following planner's problem:

$$\max_{q_r, q_s} U(q_r, q_s) \text{ subject to } q_s (V_s(q_r, q_s) - c) \geq 0, \tag{18}$$

where $U(q_r, q_s)$ is the value of the proposal per share if each shareholder acquires a private signal with probability q_s, the advisor's signal with probability q_r, and remains uninformed with probability $1-q_s-q_r$. It is given by expressions (27)–(28) in the appendix. This problem asks the following question. Suppose the planner could pick any information acquisition strategy of shareholders subject to the only constraint that the acquisition of a private signal must be incentive compatible.[23] What would be the information acquisition strategy that maximizes the informativeness of decision-making? The next proposition answers this question:

Proposition 6 (solution to the planner's problem). *The solution to the planner's problem is:*

$$(q_r, q_s) = \begin{cases} (1, 0), & \text{if } \pi \geq \pi^{**}, \\ (0, q_0^*), & \text{if } \pi \leq \pi^{**}. \end{cases}$$

Intuitively, the planner faces the trade-off between using the advisor's information and giving incentives to shareholders to acquire information privately. If the advisor's signal is sufficiently precise, the planner is satisfied with shareholders not acquiring any private signals at all. In this case, it is optimal to induce all shareholders to follow the advisor since an

[23] We do not require the second incentive compatibility condition $(q_r (V_r(q_r, q_s) - f) \geq 0)$ because for any q_r, the acquisition of the advisor's signal can be made incentive compatible by forcing the advisor to set a sufficiently low fee.

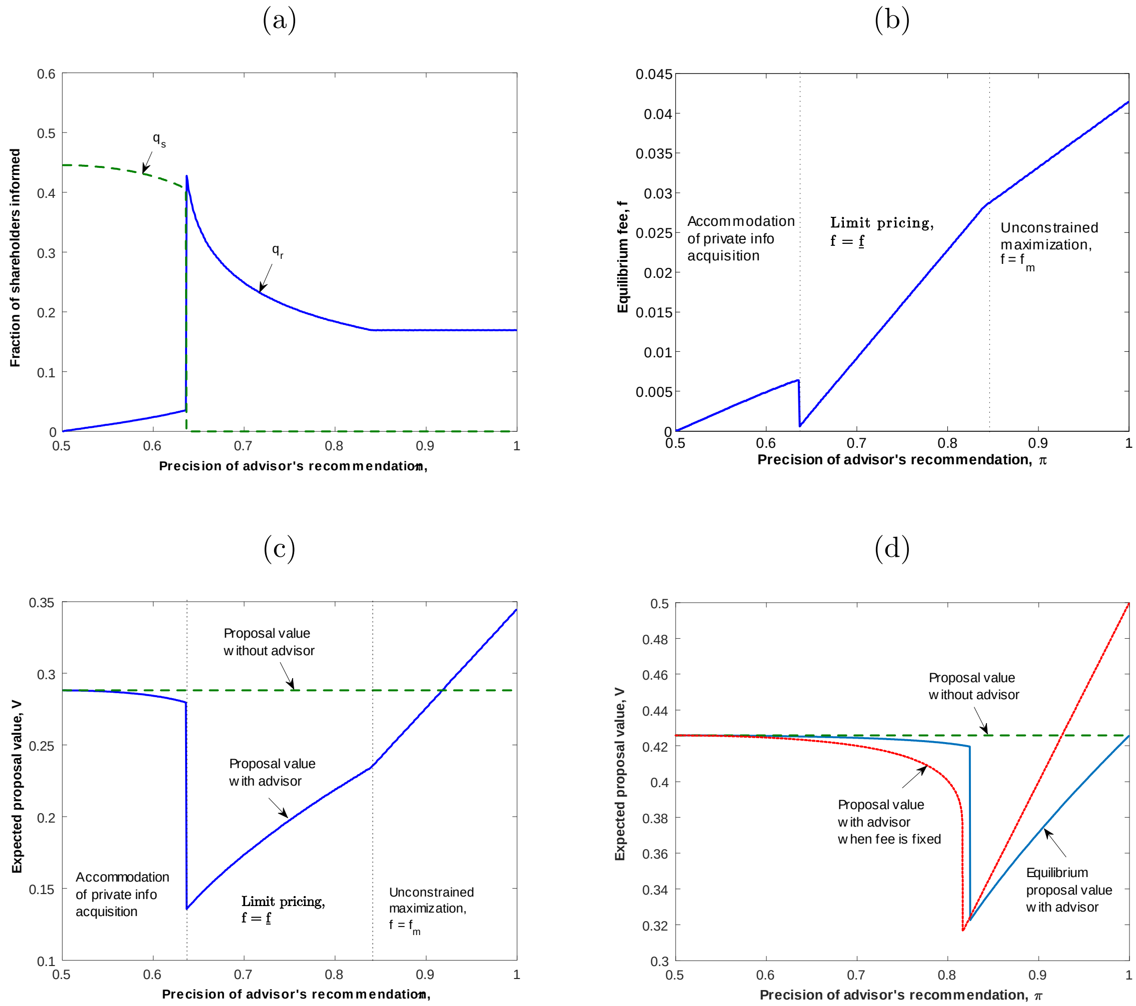

Figure 3. Equilibrium fee, information acquisition decisions, and quality of decision-making for different levels of precision of the advisor's signal. Figure 3a plots the equilibrium probability of a shareholder acquiring the advisor's recommendation (q_r) and a private signal (q_s) as functions of the precision of the advisor's signal π. Figure 3b plots the equilibrium fee set by the advisor as a function of the precision of its recommendation. Figure 3c plots the equilibrium expected value of the proposal (the solid blue line). Figure 3d plots the same figure but when the cost of private information acquisition c is half the baseline amount. As a benchmark, Figures 3c and 3d also plot the expected value of the proposal in the benchmark case of Section 2.1 (the dashed green lines). Finally, the dotted red line of Figure 3d plots the equilibrium expected value of the proposal in the model where the advisor's fee is fixed at $f = 10^{-5}$, which is the equilibrium fee ($f = \underline{f}$) at $\pi_{\min}$, where $\pi_{\min}$ is the π at which there is a switch from the equilibrium with incomplete crowding out to the equilibrium with complete crowding out of private information acquisition, i.e., the lowest π at which the equilibrium features limit pricing. The parameters are $N = 35$, $p = 0.65$, $c = 0.015$ (except Figure 3d), and $c = 0.0075$ in Figure 3d.

imperfect advisor's signal is better than voting completely uninformatively. In contrast, if the advisor's signal is not precise enough, the planner wants some shareholders to acquire private information, and in this case, as Proposition 3 shows, firm value is the highest if shareholders do not acquire the advisor's signal at all. In the former case, the correct decision is made with probability π, and in the latter case, it is made with probability π^{**}, which explains the solution above.

Comparing the equilibrium of the model to the planner's solution, we can see the role of the two sources of inefficiencies. If the advisor's signal is imprecise ($\pi < \pi^{**}$), the equilibrium features overreliance on the advisor's recommendation compared to the planner's solution because at least some shareholders will follow the advisor. In contrast, if the advisor's signal is precise ($\pi > \pi^{**}$), the equilibrium features underreliance on the advisor's information compared to the planner's solution. This latter inefficiency occurs because of the market power of the advisor. One way to implement the planner's solution would be to dictate the fee that the advisor charges to the shareholders. Proposition 6 implies that if the advisor's information is not too precise, it would be optimal to make its recommendations prohibitively expensive to deter shareholders from buying them all together. In contrast, if the advisor's information is sufficiently precise, it would be optimal to set the fee at the lowest possible level to encourage as many shareholders as possible to buy the advisor's recommendations.

Note that our results are about firm value (the probability of correct decision), and not about total welfare, which subtracts shareholders' costs of private information acquisition from firm value. In addition to the effects emphasized in this paper, the analysis of total welfare will feature an additional positive effect of the proxy advisor in that its presence reduces the total cost of private information acquisition (Nq_sc goes down). This effect is prominently featured in theories of financial intermediation of Diamond (1984) and Ramakrishnan and Thakor (1984). Whether this effect is quantitatively important in our setting depends on the degree of the free-rider problem in information acquisition. In particular, as Figure *XXX* shows, if the free-rider problem in information acquisition is sufficiently powerful, then this effect is quantitatively small, and the results about total welfare are essentially identical to the results about firm value.

4.5 The case of very dispersed ownership

For any fixed cost c of private information acquisition, Assumption 1 imposes a restriction that the number of shareholders N is not too high. In this section, we examine the role of proxy advisors for a large enough N, i.e., for the case of sufficiently dispersed ownership. Specifically, let us fix c and increase N. Proposition 1 implies that there exists a threshold $\bar{N}$ such that for any $N > \bar{N}$, the equilibrium of the benchmark model without the proxy advisor features no information acquisition at all. Intuitively, if the number of shareholders is very large, the free-rider problem in information acquisition is so severe that no shareholder finds it optimal to become informed. Clearly, because the resulting decisions are as good as pure noise, the proxy advisor's presence cannot decrease the quality of decision-making. What is more interesting, as the next proposition establishes, the proxy advisor's presence *strictly* improves the quality of decision-making in this case:

Proposition 7 (dispersed ownership). *Let $\bar{N}$ be the unique N that solves $\left(p - \frac{1}{2}\right) C_{N-1}^{\frac{N-1}{2}} 2^{1-N} = c$. Then, for any $N > \bar{N}$, firm value in the benchmark case without the advisor is strictly lower than firm value when the advisor is present.*

The reason for this result is that the proxy advisor partly internalizes the free-rider problem among shareholders in its pricing policy. Intuitively, as the number of shareholders becomes very high, the proxy advisor lowers its fee to ensure that at least some shareholders buy it. For example, in the limit case $N \to \infty$, the advisor's fee becomes infinitesimal. In turn, a lower fee encourages some shareholders to stay informed, making the decision somewhat informative even if the free rider problem in information acquisition is very severe.[24] Proposition 7 implies that the proxy advisor's presence unambiguously leads to better corporate decision-making when ownership is so dispersed that shareholders would vote uninformatively without the proxy advisor. The proxy advisor's presence can only lead to less informative decisions in cases where private information production is relevant.

[24] As shown in the appendix, in the limit case $N \to \infty$, the equilibrium fraction of shareholders who buy the advisor's recommendation converges to zero, and thus, in the limit, firm value with and without the advisor are the same (zero). However, for any finite $N > \bar{N}$, firm value is strictly higher with the advisor.

5 Analysis of regulation

In this section, we analyze three types of regulations in the context of our model. First, we study the effects of litigation pressure to subscribe to and follow the proxy advisor's recommendations. Second, we analyze regulations aimed at reducing proxy advisory fees. Finally, we examine the role of transparency.

5.1 Litigation pressure

As discussed in the introduction, the influence of proxy advisors is frequently attributed to institutions' desire to reduce the risk of litigation for their voting practices. For example, the 2003 SEC rule states that "an adviser could demonstrate that the vote was not a product of a conflict of interest if it voted client securities, in accordance with a pre-determined policy, based upon the recommendations of an independent third party." To incorporate these incentives into the model, we assume that if a shareholder subscribes to and follows the advisor's recommendation, he gets an additional payoff $\Delta > 0$, which can be interpreted as the present value of litigation costs that get saved by following the advisor.

We show that greater litigation pressure improves decision-making only if the quality of the advisor's recommendation is sufficiently high. Intuitively, an increase in litigation pressure Δ has two effects. On the one hand, it induces shareholders to vote informatively. On the other hand, it shifts the incentives from doing proprietary research to following the advisor's recommendations. The overall effect of higher Δ depends on the quality of recommendations π as follows. If π is low, then, as Section 4.4 shows, there is overreliance on the advisor's recommendation and inefficient crowding out of private information production. In this case, higher litigation pressure leads to even more inefficient crowding out of private information, which reduces firm value. In contrast, if π is high, there is underreliance on the advisor's recommendation, and in this case, greater litigation pressure improves decision-making by increasing the fraction of shareholders who follow the advisor instead of voting uninformatively. Formally, in the Online Appendix, we show that a marginal increase in Δ decreases firm value if the equilibrium features incomplete crowding out of private information acquisition, but weakly increases firm value if it features complete crowding out.

5.2 Reducing proxy advisory fees

It is frequently argued that proxy advisors, in particular ISS, have too much market power. Indeed, the industry is dominated by two players, ISS and Glass Lewis, who together control 97% of the market in terms of their clients' equity assets, with ISS controlling 61% of the market. As a result, proposals to restrict proxy advisors' market power have been widely discussed (e.g., Edelman, 2013). For example, according to the GAO (2007) report, many institutional investors believe that "increased competition could help reduce the cost [of]... proxy advisory services."

Our analysis implies that an exogenous reduction in the advisor's fees improves the quality of decision-making only if the quality of its recommendations is high enough. Formally, the Online Appendix shows that a marginal reduction in f increases firm value if equilibrium features complete crowding out of private information acquisition, but decreases firm value if equilibrium features incomplete crowding out. Intuitively, suppose that the advisor's information is not very precise, so that there is overreliance on its recommendations but some private information acquisition still occurs. In this case, lowering the advisor's fees would induce even more investors to follow its recommendations instead of doing independent research, which would be detrimental for firm value. In contrast, if the advisor's information is sufficiently precise and there is complete crowding out of private information acquisition, reducing the advisor's fees and thereby encouraging more shareholders to buy its recommendations instead of voting uninformatively is beneficial.

While formally studying competition is beyond the scope of this model, the above argument suggests that the entry of a new firm into the proxy advisory industry need not necessarily lead to more informative voting outcomes. On the one hand, the entry of a new advisor adds new information and can also increase the incumbent's incentive to invest in the quality of its recommendations. For example, Li (2016) finds that the entry of Glass Lewis alleviated the pro-management bias of ISS recommendations. On the other hand, keeping the quality of recommendations fixed, new entry also lowers the equilibrium fees, which can be harmful if there is overreliance on proxy advisory recommendations. Thus, the overall effect of entry depends on how competition affects both the price and the quality of recommendations and on the amount of new information the entrant adds.

5.3 Improving the quality of recommendations

Market participants have raised multiple concerns about the quality of proxy advisors' research, pointing out to potential conflicts of interest, inaccuracies in proxy advisory reports, and a cookie cutter approach to governace. Accordingly, several proposals have been made to improve the quality of recommendations, such as setting "qualification standards for proxy analysts" or requiring proxy advisors "to have a process that demonstrates due care towards formulating accurate voting recommendations."[25]

Our analysis shows that an exogenous increase in quality π does not necessarily lead to more informative voting oucomes.[26] Intuitively, higher recommendation quality can encourage even more shareholders to follow the advisor instead of doing independent research, which can be detrimental to firm value if the quality of recommendations is not high enough.

5.4 Disclosing the quality of recommendations

Another commonly discussed policy is to increase the transparency of proxy advisors' methodologies and procedures, to make it easier for investors to evaluate the quality of their recommendations. This includes both disclosure of potential conflicts of interest[27] and disclosure of assumptions and sources of information underlying the recommendations.[28]

To evaluate the effects of such proposals, we consider the following modification of the basic setting. The actual precision of the advisor's signal can be high or low, $\pi \in \{\pi_l, \pi_h\}$, $\pi_l < \pi_h$, with probabilities μ_l and μ_h, $\mu_h + \mu_l = 1$. Let $\bar{\pi} \equiv \mu_l \pi_l + \mu_h \pi_h$ denote the expected precision of the signal. We compare the quality of decision-making in two regimes – when the precision of the advisor's signal is publicly disclosed and when it remains unknown to the shareholders. In the first case, precision $\pi \in \{\pi_l, \pi_h\}$ is first realized and learned by all parties, and then the game proceeds exactly as in the basic model. In the second case, the timing of the game is the same as in the basic model, but both the advisor's decision about fee and shareholders' decisions which signal to acquire and how to vote are made without

[25] "A call for change in the proxy advisory industry status quo" by the Center on Executive Compensation.

[26] This automatically follows from Propositions 3 and 5, which show that firm value under $\pi > 0.5$ is lower than firm value under $\pi = 0.5$ (benchmark case) when π is low enough. See also Figure 3d.

[27] For example, the 2014 SEC Staff Legal Bulletin No. 20 requires that proxy advisors disclose potential conflicts of interest to their existing clients, but many market participants push for further regulation, which would require conflicts of interests to be disclosed to the broader public.

[28] For example, the 2010 SEC concept release on the U.S. proxy system discusses "increased disclosure regarding the extent of research involved with a particular recommendation and the extent and/or effectiveness of its controls and procedures in ensuring the accuracy of issuer data."

knowing whether $\pi = \pi_l$ or $\pi = \pi_h$.

In the Online Appendix, we develop sufficient conditions under which disclosure improves the quality of decision-making, but also demonstrate that such disclosure can sometimes be harmful. Intuitively, the benefit of disclosure is that it allows shareholders to tailor their information acquisition decisions to the quality of recommendations – shareholders do not acquire the advisor's recommendations if they learn that recommendations are of sufficiently low quality, $\pi = \pi_l$, and do not acquire private information if they learn that recommendations are of sufficiently high quality, $\pi = \pi_h$. If π_h is high enough, such tailored information acquisition is more efficient than decision-making under uncertainty about π. However, if π_h is not very high, such tailored information acquisition can decrease firm value: in this case, π_h is not high enough to improve decision-making but is sufficiently high to completely crowd out private information acquisition. This inefficient crowding out when $\pi = \pi_h$ is detrimental for firm value, and even the more efficient decision-making when $\pi = \pi_l$ is not sufficient to counteract its negative effect.

6 Endogenous quality of the advisor's recommendation

Our basic model assumes that the advisor is endowed with an informative signal, ignoring the information acquisition problem of the advisor. In this section, we enrich the basic model by assuming that the advisor decides on the precision of its signal before offering to sell it to shareholders. Specifically, suppose that the advisor can acquire signal of precision π at cost $C(\pi, t)$, which is twice continuously differentiable in π, satisfying $C\left(\frac{1}{2}, t\right) = 0$, $\frac{\partial}{\partial \pi} C\left(\frac{1}{2}, t\right) = 0$, $\frac{\partial^2}{\partial \pi^2} C(\pi, t) > 0$ for any $\pi \in \left(\frac{1}{2}, 1\right)$ and $t \in (0, \infty)$, and $\lim_{\pi \to 1} \frac{\partial}{\partial \pi} C(\pi, t) = \infty$ for any $t \in (0, \infty)$. These assumptions are intuitive: A signal that is pure noise ($\pi = \frac{1}{2}$) is costless, a perfectly informative signal is infinitely costly, and the cost function is convex in the precision level. Parameter t captures the marginal cost of making the advisor's signal more precise and satisfies $\frac{\partial^2}{\partial \pi \partial t} C(\pi, t) > 0$ for any $\pi \in \left(\frac{1}{2}, 1\right)$ and $t \in (0, \infty)$, $\lim_{t \to 0} \frac{\partial}{\partial \pi} C(\pi, t) = 0$ and $\lim_{t \to \infty} \frac{\partial C(\pi, t)}{\partial \pi} = \infty$ for any $\pi \in \left(\frac{1}{2}, 1\right)$. An example of the cost function that satisfies these restrictions is $C(\pi, t) = t\left(\frac{\pi}{1-\pi} - 1\right)^{\alpha}$ for any $\alpha > 1$. Both function $C(\pi, t)$ and parameter t are common knowledge. The timing of the model is as follows. First, the advisor decides on precision π and pays cost $C(\pi, t)$. After that, all shareholders learn the advisor's choice of π, and the sequence of actions coincides with the basic model, illustrated in Figure 1.

The next proposition establishes a result analogous to Proposition 5, the main result of the basic model:

Proposition 8. *Firm value in the presence of the advisor is strictly lower than in the benchmark case if and only if* $t < \tilde{t}$*, i.e., the advisor's information acquisition technology is sufficiently inefficient.*

The argument is as follows. The basic model implies that the advisor's presence increases firm value only if the precision of its signal is above a certain cutoff $\tilde{\pi}$. When the advisor chooses the precision endogenously, parameter t of the cost function maps into the chosen precision $\pi^*(t)$ in a monotone way, from a purely noisy signal (if $t \to \infty$) to a perfectly precise signal (if $t \to 0$). When $t = \tilde{t}$, the endogenous precision $\pi^*\left(\tilde{t}\right)$ is exactly $\tilde{\pi}$.

While our main result remains unchanged, endogenous precision of the advisor's signal can be quite important for the policy implications of the model, because regulation is likely to change the advisor's incentive to invest in information. For example, in unreported results, we show that a marginal increase in the litigation pressure parameter Δ strictly lowers the quality of the advisor's signal if the equilibrium features complete crowding out of private information acquisition. However, if the equilibrium features incomplete crowding out of private information acquisition, then a marginal increase in Δ can increase or decrease the quality of the advisor's signal, depending on the parameters of the model.

7 Discussion of assumptions and robustness

Our basic model is stylized and omits several features of the proxy advisory industry. In this section we discuss how it can be enriched to account for these features.

Correlated mistakes in private signals. The basic model assumes that private signals are independent conditional on the state, i.e., $corr\left(s_i, s_j|\theta\right) = 0$. Thus, voting mistakes of shareholders that follow private signals are uncorrelated. It is, of course, possible that shareholders could make correlated mistakes, since their signals can be based on similar sources of information. Thus, a more general model would feature private signals with positive conditional correlation, i.e., $corr\left(s_i, s_j|\theta\right) > 0$. However, as long as this correlation is imperfect, i.e., $corr\left(s_i, s_j|\theta\right) < 1$, this model would feature exactly the same trade-offs and, we conjecture, the same qualitative results.

Possibility of getting the advisor's recommendation for free. In practice, recommendations of proxy advisors sometimes leak into the press, especially on high profile cases such as contested M&A cases and proxy fights. Hence, in principle, a shareholder can sometimes "buy" the advisor's recommendation without paying the subscription fee. Since our main result holds for any positive fee f, even infinitely small (see Proposition 3), many implications of the model with possible leakage will be similar to our basic model.

It is also worth noting that many institutions subscribe to proxy advisors' services because in addition to getting the recommendation per se, the proxy advisor provides them with a detailed research report that aggregates the information necessary to make the decision and provides the arguments underlying the final binary recommendation.[29] This possibility can be captured in an extension of the model in which the advisor's research report consists of a continuous signal $r_1 \in (-\infty, \infty)$ and a binary recommendation $r_2 = I\{r_1 > 0\}$, where $I(\cdot)$ is an indicator function. While the binary recommendation can be obtained without paying the fee, a shareholder must pay the fee to get the continuous signal. Thus, the shareholder's value from subscribing to the advisor's research can be positive even if the binary recommendation is always available for free.

Possibility of acquiring both signals in equilibrium. In equilibrium of our model, no shareholder acquires both the recommendation from the advisor and a private signal. In practice, some large institutional investors both subscribe to proxy advisors' services and do their own proprietary research. The likely reason is that a shareholder's cost of producing private information and the precision of this information relative to that of the advisor's differs across proposals, depending on the type of the proposal and the shareholder's knowledge of the company. Because shareholders cannot buy the advisor's recommendations selectively, for a subset of proposals (proxy advisors sell their research on all firms and issues as a bundle), we see shareholders that both establish their own proxy research departments and subscribe to proxy advisors' services. Our model could be extended to capture this feature by introducing two proposals, such that some shareholders would pay the fee for the bundle of two recommendations but would only follow the recommendation for one of the proposals. Such a model would feature the same forces as our basic model: the advisor's presence would crowd out private information acquisition on those proposals for which shareholders would do private research without the advisor.

[29]For example, the length of research reports of ISS on high-profile M&A cases and proxy contests is about 20-30 pages, which, of course, provides much more information than a binary recommendation. See https://www.issgovernance.com/solutions/governance-advisory-services/special-situations-research/.

Complementarity between signals. Another feature of our simple binary information structure is that signals are substitutes: the value of s_i to an uninformed shareholder is higher than its value to a shareholder who buys r. With different information structures, signals can be complements: knowledge of r may increase the value of s_i to a shareholder. A model in which there is some complementarity between r and s_i will have an additional force, which goes in the direction of the advisor "crowding in" private information acquisition and, if complementarity is very strong, can outweigh the "crowding out" force we study in the paper. Since this effect of complementarity is well-known from other models of information acquisition and since there is no a priori reason why information provided by the proxy advisor and private information collected by shareholders are complements in practice, we only acknowledge it here.

8 Empirical implications

Our analysis shows that proxy advisors have a two-fold effect on the informativeness of shareholder votes, and thereby on firm value. The positive effect is that the presence (or stronger influence) of proxy advisors improves voting decisions of those shareholders who would vote uninformatively otherwise, e.g., of small shareholders who would always vote with management or vote randomly. The negative effect is that if many shareholders would invest in independent private research without the proxy advisor (e.g., shareholders with relatively large stakes in the company), the advisor's presence crowds out this private research and induces excessive conformity in shareholders' votes. Which of the two effects dominates crucially depends on the ownership structure of the company.[30]

Thus, an important implication of our paper is that, other things equal, the introduction of a proxy advisor's coverage or an exogenous shock increasing the advisor's influence has a positive effect on value in firms with sufficiently dispersed ownership, but decreases value in firms with relatively concentrated ownership if recommendations are not sufficiently precise. To test this prediction in the time series, one could look at changes in firm value after proxy advisors initiate coverage for this firm.[31] Alternatively, one could study the effect

[30] Formally, Propositions 3 and 8 show that the presence of the advisor or its stronger influence due to, e.g., stronger litigation pressure, has a positive (negative) effect on firm value in equilibrium with complete (incomplete) crowding out of private information acquisition. In turn, the proof of Proposition 7 shows that when N is large enough, $q_s = 0$ and $q_r > 0$, and hence complete crowding out is more likely when ownership is dispersed.

[31] Of course, coverage initiation is not completely exogenous, which might complicate the identification of

of regulations increasing proxy advisors' influence, such as the 2003 SEC rule discussed above and two 2004 no-action letters by the SEC, which clarified how asset managers could resolve their own conflicts of interest by relying on proxy advisors' recommendations.[32] For example, according to Sangiorgi and Spatt (2017), "these no-action letters have been very controversial because of the favorable impact upon the proxy-voting advisory firm business and the adverse societal consequences of the proxy-voting advisory firm reducing the extent of diverse information production."

Calluzzo and Dudley (2017) follow a different approach to testing the above prediction by looking at cross-sectional variation in the influence of proxy advisors: they develop a firm-level measure of ISS influence based on the propensity of the firm's shareholders to vote with ISS. They show that ISS influence is positively associated with firm value in firms with dispersed ownership, but is negatively, albeit often insignificantly, associated with firm value when ownership is more concentrated. The authors interpret this evidence as being consistent with the implications of our paper.

To test the crowding out effect more directly, one could examine shareholders' decisions to invest in independent research. One way to measure private information acquisition is to look at shareholders' votes: shareholders who acquire private information are more likely to deviate from proxy advisors' recommendations. For example, the evidence in Iliev and Lowry (2015), Ertimur, Ferri, and Oesch (2013), Larcker, McCall, and Ormazabal (2015), and Malenko and Shen (2016) suggests that shareholders are more likely to do independent research when they are large, have a large investment in the firm, and have low turnover. Another, more direct, way to measure private information acquisition by shareholders is to use the approach of Iliev, Kalodimos, and Lowry (2018), who study the downloads of firms' proxy statements and proxy-related SEC filings by large mutual fund families using the IP address data. The authors find that an institution's tendency to vote against ISS is higher when it does such independent research more.

Another prediction of our analysis is that the quality of proxy advisors' recommendations has a non-monotonic effect on firm value. Indeed, as Figure 3 demonstrates, when the quality of recommendations (π) is not very high, an increase in π allows the advisor to crowd out more private information acquisition, which decreases firm value. However, when the quality

our effects.

[32] See the "Investment Advisers Act of 1940 - Rule 206(4)-6" letter to Egan Jones and the "Investment Advisors Act of 1940 - Rule 206(4)-6" letter to ISS.

of recommendations is sufficiently high, shareholders do not invest in private information production, so a further increase in π has a positive effect on value. Regulation of the proxy advisory industry is one potential source of variation in the quality of recommendations. For example, one intention of the 2014 SEC Staff Legal Bulletin No. 20 (SLB 20) was to reduce the conflicts of interest in proxy advisors' recommendations (which could be interpreted as an increase in π) by increasing the pressure on both asset managers and proxy advisors to be vigilant about such conflicts. To measure π in the data, one could conduct a textual analysis of proxy advisors' research reports, examining the length of the report and the uniqueness of the text to measure whether the report follows a one-size-fits-all approach or takes into account firm-specific information.

9 Conclusion

In this paper, we provide a simple framework for analyzing the impact of proxy advisors on shareholder voting. In our model, a monopolistic advisor (proxy advisory firm) offers to sell its information (vote recommendations) to voters (shareholders) for a fee, and voters non-cooperatively decide whether to engage in private information production and/or buy the advisor's recommendation, and how to cast their votes. Our main results can be summarized as follows. First, the proxy advisor's presence increases firm value only if the quality of its recommendations is sufficiently high. Second, if it is not sufficiently high, there is overreliance on the advisor's recommendations relative to the degree that would maximize firm value. Finally, if the information of the advisor is very precise, there is under-reliance on its signal: because of market power, the advisor rations its information to maximize profits.

We also examine the effects of several proposals that have been put forward to regulate the proxy advisory industry. We show that increasing litigation pressure increases incentives of shareholders to vote informatively but shifts them from doing independent research to following the proxy advisor. As a consequence, increasing litigation pressure improves the quality of decision-making only if the proxy advisor's recommendations are sufficiently precise. Likewise, restricting the advisor's market power improves the quality of decision-making if its information is of high quality, but leads to greater overreliance on it and lowers firm value if it is of low-quality. Finally, higher recommendation quality and higher transparency about the quality do not unambiguously improve decision-making.

Several extensions of our model can be fruitful. First, it is natural to extend the model to allow for conflicts of interest among different shareholders and/or for a biased proxy advisor. Second, allowing for heterogeneity of shareholders in their voting power can lead to additional effects. Finally, it can be interesting to examine the optimal voting rules in this framework. Since extending the model in these directions is not straightforward, we leave them for future research.

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Appendix: Proofs

Proof of Proposition 1.

Fix probability q with which each shareholder i acquires a private signal s_i. In the Online Appendix, we prove that for any q, the equilibrium $w_s(0) = 0$, $w_s(1) = 1$, and $w_0 = \frac{1}{2}$ exists (as argued before, this is the only possible equilibrium at the voting stage because otherwise information would have zero value and acquiring it would be suboptimal).

Next, consider shareholder i's value from becoming informed. Conditional on the shareholder's private signal being $s_i = 1$, whether he is informed or not only makes a difference if the number of "for" votes among other shareholders is exactly $\frac{N-1}{2}$. Let us denote this set of events by PIV_i. In this case, by acquiring the signal, the shareholder votes "for" for sure, instead of randomizing between voting "for" and "against," so his utility from being informed is $\frac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right]$. Similarly, conditional on his private signal being $s_i = 0$, the shareholder's utility from being informed is $-\frac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]$. Overall, the shareholder's value of acquiring a private signal is

$$\begin{aligned} V(q) = &\Pr(s_i = 1)\Pr(PIV_i|s_i = 1)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] \\ &-\Pr(s_i = 0)\Pr(PIV_i|s_i = 0)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]. \end{aligned}$$

By the symmetry of the setup and strategies, $\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] = -\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]$ and $\Pr(PIV_i|s_i = 1) = \Pr(PIV_i|s_i = 0)$, so we get

$$\begin{aligned} V(q) &= \tfrac{1}{2}\Pr(PIV_i|s_i = 1)\,\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] \\ &= \tfrac{1}{2}\Pr(PIV_i|s_i = 1)\left(\Pr\left[\theta = 1|s_i = 1, PIV_i\right] - \Pr\left[\theta = 0|s_i = 1, PIV_i\right]\right) \\ &= \Pr\left[\theta = 1, PIV_i, s_i = 1\right] - \Pr\left[\theta = 0, PIV_i, s_i = 1\right] = \tfrac{1}{2}p\Pr\left[PIV_i|\theta = 1\right] - \tfrac{1}{2}(1-p)\Pr\left[PIV_i|\theta = 0\right] \end{aligned}$$

Conditional on $\theta = 1$, other shareholders make their voting decisions independently and vote "for" with probability $qp + \frac{1}{2}(1-q) = \frac{1}{2} + q\left(p - \frac{1}{2}\right)$. Hence,

$$\Pr\left[PIV_i|\theta = 1\right] = C_{N-1}^{\frac{N-1}{2}}\left(\frac{1}{2} + q(p - \frac{1}{2})\right)^{\frac{N-1}{2}}\left(\frac{1}{2} - q(p - \frac{1}{2})\right)^{\frac{N-1}{2}}.$$

Noting that $\Pr\left[PIV_i|\theta = 1\right] = \Pr\left[PIV_i|\theta = 0\right]$ gives (6). Note that $V(q)$ decreases in q. Since $P\left(x, N-1, \frac{N-1}{2}\right)$ decreases in N for any x, it follows that $V(q)$ decreases in N.

In deciding whether to acquire the private signal, shareholder i compares the expected value of his signal $V(q)$ with cost c. Since $V(q)$ is strictly decreasing in q, there are three possible cases. If $c < \underline{c} \equiv V(1)$, then each shareholder acquires information regardless of q. Hence, in the unique equilibrium all shareholders acquire private signals: $q^* = 1$. If $c > \bar{c} \equiv V(0)$, then each shareholder is better off not acquiring information regardless of q. Hence, in the unique equilibrium all shareholders remain uninformed: $q^* = 0$. Finally, if $c \in [\underline{c}, \bar{c}]$, then q^* is given as the solution to $V(q^*) = c$. Plugging (6) and rearranging the terms, we get (7).

Finally, we derive the equilibrium firm value given q_0^* :

$$V_0 = \Pr(\theta = 1) \sum_{k=\frac{N+1}{2}}^{N} P(q_0^* p + \tfrac{1-q_0^*}{2}, N, k) - \Pr(\theta = 0) \sum_{k=\frac{N+1}{2}}^{N} P(q_0^*(1-p) + \tfrac{1-q_0^*}{2}, N, k)$$
$$= \tfrac{1}{2} \sum_{k=\frac{N+1}{2}}^{N} P(\tfrac{1}{2} + \Lambda, N, k) - \tfrac{1}{2} \sum_{k=\frac{N+1}{2}}^{N} P(\tfrac{1}{2} + \Lambda, N, N-k)] = \sum_{k=\frac{N+1}{2}}^{N} P(\tfrac{1}{2} + \Lambda, N, k) - \tfrac{1}{2},$$

where we used $\sum_{k=0}^{N} P(q, N, k) = 1$.

Proof of Proposition 2.

Let us prove that there is no equilibrium in which a shareholder acquires both signals with positive probability. By contradiction, suppose such an equilibrium exists and consider a shareholder with both signals, r and s_i. Consider a realization $r = 1$ and $s_i = 0$. There are three possibilities: $w_{rs}(1,0) = 1$, $w_{rs}(1,0) = 0$, and $w_{rs}(1,0) \in (0,1)$. First, if $w_{rs}(1,0) = 1$, then it must be that $w_{rs}(1,1) = 1$ because the shareholder's posterior that $\theta = 1$ is strictly higher in this case. By symmetry, $w_{rs}(0,1) = 1 - w_{rs}(1,0) = 0$. In turn, $w_{rs}(0,1) = 0$ implies $w_{rs}(0,0) = 0$, since the shareholder's posterior that $\theta = 1$ is strictly lower in this case. It follows that $v_i = r$, and hence the shareholder would be better off if he acquired only the advisor's signal. Second, if $w_{rs}(1,0) = 0$, then it must be that $w_{rs}(0,0) = 0$. By symmetry, $w_{rs}(0,1) = 1 - w_{rs}(1,0) = 1$, and hence $w_{rs}(1,1) = 1$. It follows that $v_i = s_i$, and hence the shareholder would be better off if he only acquired the private signal. Finally, if $w_{rs}(1,0) \in (0,1)$, then by symmetry $w_{rs}(0,1) = 1 - w_{rs}(1,0) \in (0,1)$. Hence, when $r \neq s_i$, the shareholder is indifferent between voting $v_i = r$ and $v_i = s_i$. Hence, the shareholder would be better off if he only acquired one signal of the two.

The arguments in the text preceding Proposition 2 complete the proof. In the Online Appendix, we derive the condition under which equilibrium $w_s(s_i) = s_i$, $w_r(r) = r$, and $w_0 = \frac{1}{2}$ will exist for any possible sub-game. However, whenever this condition is violated, this sub-game features zero value of recommendation of the advisor, and hence is not reached on equilibrium path if $q_r > 0$.

Proof of Lemma 1. To prove the lemma, we derive the necessary and sufficient conditions for each type of equilibrium to exist.

1. Equilibrium with only private information acquisition. Consider the case of $q_r = 0$. In this case, a shareholder's choice between buying a private signal and staying uninformed is identical to the situation in which there is no advisor, covered in Proposition 1. Hence, $q_s = q_0^* \in (0,1)$. Pair $(q_r, q_s) = (0, q_0^*)$ is an equilibrium if and only if no shareholder would be better off deviating to buying recommendation from the advisor: $V_r(0, q_0^*) \leq f$. Since $\Omega_1(0, q_0^*) = \Omega_2(0, q_0^*) = \frac{c}{p-0.5}$ (the latter by indifference $V_s(0, q_0^*) = c$), $V_r(0, q_0^*) = \frac{\pi-0.5}{p-0.5}c$. Hence, $V_r(0, q_0^*) \leq f$ is equivalent to $f \geq \bar{f} = \frac{\pi-0.5}{p-0.5}c$.

2. Equilibrium with complete crowding out of private information acquisition. Consider the case of $q_s = 0$. Then it must be that $q_r \in (0,1)$. Indeed, it cannot be that $q_r = 0$, since if $q_r = 0$, then the value of acquiring a private signal is $V_s(0,0) = \bar{c} > c$ by Assumption 1, so a shareholder would be better off deviating to acquiring a private signal. It also cannot be that $q_r = 1$, since in that case no shareholder would be pivotal, so $V_r(1,0) = 0 < f$ for any $f > 0$. Thus, a shareholder would be better off deviating to staying uninformed. For $q_s = 0$ and $q_r \in (0,1)$ to constitute an equilibrium, it is necessary and sufficient that $V_s(q_r, 0) \le c$ and $V_r(q_r, 0) = f$. When $q_s = 0$, the probabilities of being pivotal are:

$$\Omega_1(q_r, 0) = P\left(\frac{1+q_r}{2}, N-1, \frac{N-1}{2}\right) = P\left(\frac{1-q_r}{2}, N-1, \frac{N-1}{2}\right) = \Omega_2(q_r, 0) \equiv \Omega_r(q_r). \quad (19)$$

Eq. $V_r(q_r, 0) = f$ yields $\Omega_r(q_r) = \frac{f}{\pi - 0.5}$. Equating to (19), we obtain that q_r is given by (13), which lies in $(0,1)$ if $f < C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(\pi - \frac{1}{2}\right)$. Otherwise, no solution exists. Plugging $\Omega_r(q_r) = \frac{f}{\pi-0.5}$ into $c \ge V_s(q_r, 0)$, we obtain $f \le \frac{2\pi-1}{2p-1}c$. Note that

$$C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(\pi - \frac{1}{2}\right) > \frac{2\pi-1}{2p-1}c \Leftrightarrow \frac{1}{4} > \left(\frac{c}{\left(p-\frac{1}{2}\right)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}},$$

which is satisfied by Assumption 1. Hence, the equilibrium with complete crowding out of private information acquisition exists if and only if $f \le \bar{f}$.

3. Equilibrium with incomplete crowding out of private information acquisition. Consider the case of $q_s > 0$. If $q_r + q_s < 1$ in equilibrium, then a shareholder must be indifferent between acquiring r, acquiring s_i, and staying uninformed. Hence, q_s and q_r must satisfy $V_s(q_r, q_s) = c$ and $V_r(q_r, q_s) = f$, which yields a system of linear equations for Ω_1 and Ω_2:

$$\begin{cases} \pi\Omega_1 + (1-\pi)\Omega_2 = \frac{c}{p-0.5} \\ \pi\Omega_1 - (1-\pi)\Omega_2 = 2f \end{cases} \Leftrightarrow \Omega_1 = \frac{f + \frac{c}{2p-1}}{\pi} \text{ and } \Omega_2 = \frac{\frac{c}{2p-1} - f}{1-\pi}. \quad (20)$$

Since the second equality implies $f \le \frac{c}{2p-1}$, this system is equivalent to the following system of equations for q_r and q_s:

$$\begin{aligned} \left(\tfrac{1}{2}q_r + \left(p-\tfrac{1}{2}\right)q_s\right)^2 &= \tfrac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}, \\ \left(\tfrac{1}{2}q_r - \left(p-\tfrac{1}{2}\right)q_s\right)^2 &= \tfrac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi) C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}. \end{aligned} \quad (21)$$

It has a solution if and only if the right-hand sides of both equations are non-negative, i.e., if $f \in \left[\underline{f}_1, 2^{1-N}\pi C_{N-1}^{\frac{N-1}{2}} - \frac{c}{2p-1}\right]$, where

$$\underline{f}_1 \equiv \frac{c}{2p-1} - 2^{1-N}(1-\pi)C_{N-1}^{\frac{N-1}{2}}, \quad (22)$$

in which case there are two solutions:

1. Solution with $q_r \leq (2p-1)\, q_s$, denoted (q_r^a, q_s^a):

$$
\begin{aligned}
q_r^a &= \sqrt{\frac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}} - \sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi) C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}}, \\
q_s^a &= \frac{1}{2p-1}\left(\sqrt{\frac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}} + \sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi) C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}}\right).
\end{aligned}
\tag{23}
$$

2. Solution with $q_r \geq (2p-1)\, q_s$, denoted $\left(q_r^b, q_s^b\right)$:

$$
\begin{aligned}
q_r^b &= \sqrt{\frac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}} + \sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi) C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}}, \\
q_s^b &= \frac{1}{2p-1}\left(\sqrt{\frac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}} - \sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi) C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}}\right).
\end{aligned}
\tag{24}
$$

Each solution is an equilibrium if and only if it satisfies $q_r > 0$, $q_s > 0$, and $q_r + q_s < 1$. Each solution satisfies $(q_r, q_s) > 0$ if and only if $\frac{f+\frac{c}{2p-1}}{\pi} < \frac{\frac{c}{2p-1}-f}{1-\pi} \Leftrightarrow f < \bar{f}$. Also, since $p \in \left(\frac{1}{2}, 1\right)$, it is easy to see that $q_r^b + q_s^b \leq q_r^a + q_s^a$.

If $q_r + q_s = 1$ in equilibrium, then a shareholder must be indifferent between acquiring r and s_i and weakly prefer this over staying uninformed. Hence, q_s and q_r must satisfy $V_s(q_r, q_s) - c = V_r(q_r, q_s) - f \geq 0$ and $q_s + q_r = 1$. The former implies

$$
\left(p - \frac{1}{2}\right)(\pi\Omega_1 + (1-\pi)\,\Omega_2) - c = \frac{1}{2}(\pi\Omega_1 - (1-\pi)\,\Omega_2) - f \equiv \psi \geq 0. \tag{25}
$$

For any ψ, these two equations lead to a system identical to (20)$\Leftrightarrow$(21), but with $c+\psi$ and $f+\psi$ instead of c and f. It has a solution if and only if the right-hand sides of both equations are positive. In that case, it has two solutions, analogous to (23) and (24), and given by (46) and (47) in the Online Appendix.

To prove the lemma, we show the following sequence of three auxiliary claims, which are proved in the Online Appendix.

1. Claim 1: If $f \geq \bar{f}$, then there is no equilibrium $(q_r, q_s) > 0$.

2. Claim 2: If $\frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\underline{f}_1+\frac{c}{2p-1}}{\pi C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}} \leq 1$, there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}_1, \bar{f}\right)$, where $\underline{f}_1$ is given by (22).

3. Claim 3: If $\frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\underline{f}_1+\frac{c}{2p-1}}{\pi C_{\frac{N-1}{2}}^{N-1}}\right)^{\frac{2}{N-1}}} > 1$, there exists $\underline{f}_2 \geq \underline{f}_1$ such that there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}_2, \bar{f}\right)$.

Combining Claims 2 and 3, we conclude that there exists an equilibrium $(q_r, q_s) > 0$ if and only if $f \in [\underline{f}, \bar{f})$, where

$$\underline{f} \equiv \begin{cases} \underline{f}_1 & \text{if } \frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\underline{f}_1 + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \leq 1 \\ \underline{f}_2 & \text{otherwise,} \end{cases} \tag{26}$$

where $\underline{f}_1$ is given by (22) and $\underline{f}_2$ is defined in Claim 3, respectively. Combining this condition and the conditions of existence of equilibrium with only private information acquisition and equilibrium with complete crowding out of private information acquisition, we get the statement of the lemma.

Proof of Proposition 3.

Consider an equilibrium defined by pair q_s and q_r. Let $U(q_r, q_s)$ denote the corresponding expected value of a proposal per share. By definition,

$$\begin{aligned} U(q_r, q_s) = \mathbb{E}[u(1,\theta)\,d] &= \tfrac{1}{2}\mathbb{E}\left[\textstyle\sum_{j=1}^N v_j > \frac{N-1}{2}|\theta = 1\right] - \tfrac{1}{2}\mathbb{E}\left[\textstyle\sum_{j=1}^N v_j > \frac{N-1}{2}|\theta = 0\right] \\ &= \tfrac{1}{2}\pi\left(\textstyle\sum_{k=\frac{N+1}{2}}^N P(p_a, N, k) - \sum_{k=\frac{N+1}{2}}^N P(1-p_a, N, k)\right) \\ &\quad + \tfrac{1}{2}(1-\pi)\left(\textstyle\sum_{k=\frac{N+1}{2}}^N P(p_d, N, k) - \sum_{k=\frac{N+1}{2}}^N P(1-p_d, N, k)\right), \end{aligned}$$

where

$$\begin{aligned} p_a &\equiv \Pr(v_i = \theta | r = \theta) = q_r + q_s p + \tfrac{1-q_r-q_s}{2} = \tfrac{1}{2} + \tfrac{1}{2}q_r + \left(p - \tfrac{1}{2}\right)q_s, \\ p_d &\equiv \Pr(v_i = \theta | r \neq \theta) = q_s p + \tfrac{1-q_r-q_s}{2} = \tfrac{1}{2} - \tfrac{1}{2}q_r + \left(p - \tfrac{1}{2}\right)q_s, \end{aligned} \tag{27}$$

are the probabilities that a random shareholder votes correctly conditional on the proxy advisor's recommendation being correct and incorrect, respectively. Using $P(q, N, k) = P(1-q, N, N-k)$ and $\sum_{k=0}^N P(q, N, k) = 1$, the above expression simplifies to

$$U(q_r, q_s) = \sum_{k=\frac{N+1}{2}}^N \left(\pi P(p_a, N, k) + (1-\pi) P(p_d, N, k)\right) - \frac{1}{2}. \tag{28}$$

Proof of part 1. Note that the probability of a shareholder being pivotal in equilibrium with incomplete crowding out does not exceed that in the benchmark case:

$$\begin{aligned} &\pi P\left(p_a, N-1, \tfrac{N-1}{2}\right) + (1-\pi) P\left(p_d, N-1, \tfrac{N-1}{2}\right) = \\ &= \pi\Omega_1(q_r, q_s) + (1-\pi)\Omega_2(q_r, q_s) \geq \tfrac{2c}{2p-1}. \end{aligned}$$

Indeed, it exactly equals $\frac{2c}{2p-1}$ if $q_s + q_r < 1$ based on (20), and equals $\frac{2(c+\psi)}{2p-1} \geq \frac{2c}{2p-1}$ if $q_s + q_r = 1$, where $\psi \geq 0$ is given by (25). Consider the following optimization problem:

$$\begin{aligned} &\max\nolimits_{p_a, p_d} \textstyle\sum_{k=\frac{N+1}{2}}^N \left(\pi P(p_a, N, k) + (1-\pi) P(p_d, N, k)\right) - \tfrac{1}{2} \\ &\text{s.to } \pi P\left(p_a, N-1, \tfrac{N-1}{2}\right) + (1-\pi) P\left(p_d, N-1, \tfrac{N-1}{2}\right) \geq \tfrac{2c}{2p-1} \end{aligned} \tag{29}$$

This optimization problem chooses the probabilities of a correct vote, p_a and p_d, that maximize firm value subject to the "budget constraint" that the probability that a shareholder is pivotal, implied by p_a and p_d, cannot be below $\frac{2c}{2p-1}$, i.e., that in the benchmark case. In what follows, we show that this optimization problem is solved by $p_a = p_d = \frac{1}{2} + q_0^* \left(p - \frac{1}{2}\right)$, i.e., the same as in the benchmark case. Let $x_a \equiv P(p_a, N-1, \frac{N-1}{2})$ and $x_d \equiv P(p_d, N-1, \frac{N-1}{2})$. Let us define function $\varphi(x) \in (\frac{1}{2}, 1)$ as the higher root of $x = P(\varphi(x), N-1, \frac{N-1}{2}) = C_{N-1}^{\frac{N-1}{2}} (\varphi(x)(1-\varphi(x)))^{\frac{N-1}{2}}$:

$$\varphi(x) \equiv \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{x}{C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}. \tag{30}$$

Note that $p_a > \frac{1}{2}$ and hence $p_a = \varphi(x_a)$. If $p_d > \frac{1}{2}$, then $p_d = \varphi(x_d)$, and if $p_d < \frac{1}{2}$, then $p_d = 1 - \varphi(x_d)$. First, consider all equilibria with $p_d > \frac{1}{2}$. Then, we can rewrite (29) as:

$$\begin{array}{c} \max_{x_a, x_d} \sum_{k=\frac{N+1}{2}}^{N} (\pi P(\varphi(x_a), N, k) + (1-\pi) P(\varphi(x_d), N, k)) - \frac{1}{2} \\ \text{s.t. } \pi x_a + (1-\pi) x_d \geq \frac{2c}{2p-1}, \end{array} \tag{31}$$

Auxiliary Lemma A1 at the end of the Appendix shows that function $f(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$ is strictly decreasing in x. Thus, the constraint in (31) is binding. Auxiliary Lemma A1 also shows that $f(x)$ is strictly concave in x. Thus, by Jensen's inequality, for any x_a, x_d such that $\pi x_a + (1-\pi) x_d = \frac{2c}{2p-1}$, we have

$$\pi f(x_a) + (1-\pi) f(x_d) < f(\pi x_a + (1-\pi) x_d) = f\left(\frac{2c}{2p-1}\right) = \pi f\left(\frac{2c}{2p-1}\right) + (1-\pi) f\left(\frac{2c}{2p-1}\right).$$

Therefore, there is a unique solution to the maximization problem (31), given by $x_a = x_d = \frac{2c}{2p-1}$, which gives firm value in the benchmark case. Hence, for any equilibrium with incomplete crowding out and $p_d > \frac{1}{2}$, firm value is strictly lower than in the benchmark case. Next, consider all equilibria with $p_d < \frac{1}{2}$. Note that $\sum_{k=\frac{N+1}{2}}^{N} P(1-q, N, k) = \sum_{k=\frac{N+1}{2}}^{N} P(q, N, N-k) = 1 - \sum_{k=\frac{N+1}{2}}^{N} P(q, N, k)$. In addition, $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) = -\sum_{k=0}^{\frac{N-1}{2}} P_q(q, N, k) > 0$ for $q \geq \frac{1}{2}$ because $P_q(q, N, k) = P(q, N, k) \frac{k - Nq}{q(1-q)} < 0$ for any $k < \frac{N}{2}$ and $q \geq \frac{1}{2}$. Since $\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2}, N, k\right) = \frac{1}{2}$, it follows that $\sum_{k=\frac{N+1}{2}}^{N} P(1-q, N, k) < \frac{1}{2} < \sum_{k=\frac{N+1}{2}}^{N} P(q, N, k)$ for $q > \frac{1}{2}$. Therefore,

$$\begin{array}{c} \sum_{k=\frac{N+1}{2}}^{N} \begin{pmatrix} \pi P(p_a, N, k) \\ +(1-\pi) P(p_d, N, k) \end{pmatrix} - \frac{1}{2} = \sum_{k=\frac{N+1}{2}}^{N} \begin{pmatrix} \pi P(\varphi(x_a), N, k) \\ +(1-\pi) P(1-\varphi(x_d), N, k) \end{pmatrix} - \frac{1}{2} \\ < \sum_{k=\frac{N+1}{2}}^{N} (\pi P(\varphi(x_a), N, k) + (1-\pi) P(\varphi(x_d), N, k)) - \frac{1}{2}, \end{array}$$

and the last expression, subject to the constraint in (31), has already been shown to be below firm value in the benchmark case. Hence, the quality of decision-making in any equilibrium with incomplete crowding out is strictly lower than in the benchmark case.

Proof of part 2. Next, we prove the second part of the proposition. In the equilibrium with complete crowding out of private information acquisition, we have

$$\begin{aligned}
p_a &= \tfrac{1}{2} + \tfrac{1}{2} q_r = \tfrac{1}{2} + \sqrt{\tfrac{1}{4} - \left(\frac{f}{(\pi - \frac{1}{2}) C_{N-1}^{\frac{N-1}{2}}} \right)^{\frac{2}{N-1}}}, \\
p_d &= \tfrac{1}{2} - \tfrac{1}{2} q_r = \tfrac{1}{2} - \sqrt{\tfrac{1}{4} - \left(\frac{f}{(\pi - \frac{1}{2}) C_{N-1}^{\frac{N-1}{2}}} \right)^{\frac{2}{N-1}}}.
\end{aligned} \tag{32}$$

Since $p_d = 1 - p_a$, we can rewrite firm value as

$$U = \pi \sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) + (1-\pi) \sum_{k=0}^{\frac{N-1}{2}} P(p_a, N, k) - \frac{1}{2} = \frac{1}{2} - \pi + (2\pi - 1) \sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k). \tag{33}$$

By (7) and (8), the expected value in the benchmark case without the advisor is given by $U = \sum_{k=\frac{N+1}{2}}^{N} P(p^*, N, k) - \frac{1}{2}$, where $p^* = \frac{1}{2} + q_0^* \left(p - \frac{1}{2}\right)$. Firm value is higher with the advisor than without it if and only if

$$(2\pi - 1) \sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) - \pi > \sum_{k=\frac{N+1}{2}}^{N} P(p^*, N, k) - 1. \tag{34}$$

In the Online Appendix, we show that the left-hand side of (34) is strictly increasing in π, that (34) is violated for $\pi \to \frac{1}{2} + \frac{f}{c}\left(p - \frac{1}{2}\right)$ and is satisfied for $\pi \to 1$. By monotonicity, there exists a unique $\pi^*(f) \in (\frac{1}{2} + \frac{f}{c}(p - \frac{1}{2}), 1)$ such that the advisor's presence increases firm value if and only if $\pi \geq \pi^*(f)$.

Proof of Lemma 2. We start by defining $\hat{c}$ in the statement of the lemma. Consider (q_r^a, q_s^a) given by (23) and define

$$S(f, c) \equiv q_r^a + q_s^a = \frac{2p}{2p-1} \sqrt{\frac{1}{4} - \left(\frac{f + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}} \right)^{\frac{2}{N-1}}} + \frac{2(1-p)}{2p-1} \sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1} - f}{(1-\pi) C_{N-1}^{\frac{N-1}{2}}} \right)^{\frac{2}{N-1}}}.$$

Consider the following function of c:

$$V(c) \equiv \max_{f \in \left[\underline{f}_1(c), \bar{f}(c)\right]} S(f, c), \tag{35}$$

where $\underline{f}_1(c) = \frac{c}{2p-1} - 2^{1-N}(1-\pi) C_{N-1}^{\frac{N-1}{2}}$ and $\bar{f}(c) = \frac{2\pi-1}{2p-1} c$, as defined before. In the Online Appendix, we show that $V(c)$ is strictly decreasing in c. Note also that when $c = \underline{c}$, defined in (7), then $S\left(\bar{f}(\underline{c}), \underline{c}\right) = 1$. Hence, $V(\underline{c}) \geq 1$. In addition, when $c = \bar{c}$, defined in (7), then $\bar{f}(\bar{c}) = \underline{f}_1(\bar{c})$, and hence $V(\bar{c}) = S\left(\bar{f}(\bar{c}), \bar{c}\right) = 0$. Hence, there exists a unique $\hat{c} \in [\underline{c}, \bar{c})$ at which $V(\hat{c}) = 1$, and $V(c) < 1$ for any $c \in (\hat{c}, \bar{c})$. To sum up, we define $\hat{c} \equiv V^{-1}(1)$, where $V(c)$ is given by (6).

Suppose that $c \in (\hat{c}, \bar{c})$. Then, $\frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\underline{f}_1 + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} = S\left(\underline{f}_1, c\right) \leq V(c) < 1$, and hence $\underline{f} = \underline{f}_1$ according to (26). According to Claim 2 in the proof of Lemma 1, there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}, \bar{f}\right)$. Let us find all such equilibria. Since $V(c) < 1$, then $q_r^a + q_s^a < 1$ for any $f \in \left[\underline{f}, \bar{f}\right)$. Therefore, $q_r^b + q_s^b \leq q_r^a + q_s^a < 1$. In addition, $(q_r^a, q_s^a) > 0$ and $\left(q_r^b, q_s^b\right) > 0$ because $f < \bar{f}$. Thus, both equilibria (23) and (24) exist. Since $q_r^1(\psi) + q_s^1(\psi)$ is strictly decreasing in ψ and $q_r^1(0) + q_s^1(0) = q_r^a + q_s^a < 1$, we have $q_r^2(\psi) + q_s^2(\psi) \leq q_r^1(\psi) + q_s^1(\psi) < 1$ for any $\psi \geq 0$, where $\left(q_r^i(\psi), q_s^i(\psi)\right)$, $i = 1, 2$, represent potential solutions for $q_r + q_s = 1$ and are given by (46) and (47) in the Online Appendix. Therefore, there is no equilibrium with $q_s + q_r = 1$ when $f \in \left[\underline{f}, \bar{f}\right)$. Thus, in addition to equilibrium with complete crowding out, there exist exactly two other equilibria when $f \in \left[\underline{f}, , \bar{f}\right)$, and these equilibria feature incomplete crowding out with $q_r + q_s < 1$: (23) with $q_r^a \leq (2p-1)\, q_s^a$ and (24) with $q_r^b \geq (2p-1)\, q_s^b$.

The expected welfare of a shareholder is the expected per-share value of the proposal, $U(q_r, q_s)$, given by (28), minus the expected information acquisition cost:

$$W(q_r, q_s) = \sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(p_a, N, k) + (1-\pi)\, P(p_d, N, k)\right) - \frac{1}{2} - q_r f - q_s c. \tag{36}$$

In the Online Appendix, we rank these three equilibria in shareholder welfare and show that the equilibrium with incomplete crowding out of private information acquisition and $q_r < (2p-1)\, q_s$ has the highest shareholder welfare, followed by the equilibrium with incomplete crowding out of private information acquisition and $q_r > (2p-1)\, q_s$, which is followed by the equilibrium with complete crowding out of private information acquisition.

Proof of Proposition 4. The first three statements of the proposition follow directly from Lemma 1 and Lemma 2. Note also that given $c > \hat{c}$ in Assumption 2, we have $\underline{f} = \underline{f}_1$, where $\underline{f}_1$ is given by (22). We next prove the last statement of the proposition. First, consider $f < \underline{f}$. From (13) q_r is strictly decreasing in f, and from (33) firm value is strictly increasing in p_a (and hence, in q_r, as $p_a = \frac{1}{2} + \frac{1}{2} q_r$). Hence, firm value is strictly decreasing in f for $f < \underline{f}$. Second, consider $f \in [\underline{f}, \bar{f})$. In this range, firm value equals $\pi f(x_a) + (1-\pi)\, f(x_d)$, where $f(x) = \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$, $x_a = \frac{f + \frac{c}{2p-1}}{\pi}$ and $x_d \equiv \frac{\frac{c}{2p-1} - f}{1-\pi}$. Differentiating firm value in fee f yields $f'(x_a) - f'(x_b) = -\int_{x_a}^{x_b} f''(x)\, dx > 0$ by $x_a < x_d$ (follows from $f < \overline{f}$) and $f''(\cdot) < 0$ (follows from Auxilliary Lemma A1). Hence, firm value is strictly increasing in f for $f \in [\underline{f}, \bar{f})$. Finally, if $f \geq \overline{f}$, then firm value equals V_0, so it is unaffected by f.

Proof of Proposition 5. Consider the first statement of the proposition. The first part of Proposition 3 implies that if equilibrium features incomplete crowding out, then firm value is strictly lower than in the benchmark case. Hence, to find the conditions under which firm value is higher with the advisor, it is sufficient to find conditions under which the advisor sets fee in a way that crowds out private information acquisition. In case of complete crowding out, there is a one-to-one correspondence between the fee f set by the advisor and the fraction $q_r^H(f)$ buying its recommendation, where $q_r^H(f)$ is given by (13). Moreover, recall that the value of the advisor's signal to a shareholder is given by $V_r(q_r, 0) = (\pi - \frac{1}{2}) P(\frac{1+q_r}{2}, N-1, \frac{N-1}{2})$ and must be equal to f. Thus, in this case, the advisor's problem is equivalent to maximizing $q_r V_r(q_r, 0)$ over q_r. Hence, instead of choosing fee f and maximizing $f q_r^H(f)$, the advisor can choose q_r and maximize $\eta(q_r) = P(\frac{1+q_r}{2}, N-1, \frac{N-1}{2}) q_r = C_{N-1}^{\frac{N-1}{2}} \left(\frac{(1+q)(1-q)}{4}\right)^{\frac{N-1}{2}} q$. Note that

$$\frac{d\eta}{dq} = const \times \frac{d}{dq}\left[q\left(1-q^2\right)^{\frac{N-1}{2}}\right] = const \times \left(1-q^2\right)^{\frac{N-3}{2}}\left(1-Nq^2\right).$$

Hence, $\eta(q)$ is inverted U-shaped in q with a maximum at $q_m = \frac{1}{\sqrt{N}}$. The optimal fraction $q_m = \frac{1}{\sqrt{N}}$ translates into the optimal fee given by (15). The fact that $\eta(q)$ is inverse U-shaped in q implies that under complete crowding out, the advisor's revenue is maximized at $f = f_m$ and is monotonically decreasing as f gets farther from f_m in both directions. Hence, the optimal pricing strategy of the advisor if $f_m \geq \underline{f}$ is to either set $f = \underline{f} - \varepsilon$, $\varepsilon \to 0$, or to choose the fee that maximizes its revenue under incomplete crowding out, where $\underline{f} = \underline{f}_1$ is given by (22). In the second case, firm value is lower than in the benchmark case according to Proposition 3. In the first case, firm value converges to firm value under complete crowding out and $f = \underline{f}_1$. In the Online Appendix, we show that when $f = \underline{f}_1$, firm value in equilibrium with complete crowding out is strictly lower than in equilibrium with incomplete crowding out, which (by Proposition 3) is in turn lower than firm value in the benchmark case. Therefore, the only case where firm value can be higher than in the benchmark case is when $f_m < \underline{f} = \underline{f}_1$, so that the advisor chooses fee f_m. The constraint $f_m < \underline{f}_1$ can be simplified to

$$\pi > \hat{\pi} \equiv \frac{1}{2}\left(1 + \frac{C_{N-1}^{\frac{N-1}{2}} 2^{1-N} - \frac{2c}{2p-1}}{C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(1 - \left(\frac{N-1}{N}\right)^{\frac{N-1}{2}}\right)}\right).$$

If each shareholder acquires the advisor's signal with probability q_r and remains uninformed otherwise, expected firm value is given by

$$\begin{aligned} V^*(\pi, q_r) &= \Pr(\theta = 1) \textstyle\sum_{k=\frac{N+1}{2}}^{N} \left[\pi P\left(q_r + \frac{1-q_r}{2}, N, k\right) + (1-\pi) P\left(\frac{1-q_r}{2}, N, k\right)\right] \\ &\quad - \Pr(\theta = 0) \textstyle\sum_{k=\frac{N+1}{2}}^{N} \left[\pi P\left(\frac{1-q_r}{2}, N, k\right) + (1-\pi) P\left(q_r + \frac{1-q_r}{2}, N, k\right)\right] \\ &= (\pi - \tfrac{1}{2}) \textstyle\sum_{k=\frac{N+1}{2}}^{N} \left[P\left(\frac{1+q_r}{2}, N, k\right) - P\left(\frac{1-q_r}{2}, N, k\right)\right] \\ &= (2\pi - 1)\left[\textstyle\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1+q_r}{2}, N, k\right) - \frac{1}{2}\right]. \end{aligned} \tag{37}$$

Plugging in $q_r = \frac{1}{\sqrt{N}}$ in (37) and comparing it with V_0, we get

$$\begin{aligned} (2\pi - 1)\,&[\textstyle\sum_{k=\frac{N+1}{2}}^{N} P(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k) - \frac{1}{2}] > V_0 = \textstyle\sum_{k=\frac{N+1}{2}}^{N} P(\frac{1}{2} + \Lambda, N, k) - \frac{1}{2} \\ &= \pi^{**} - \tfrac{1}{2} \Leftrightarrow \pi > \tilde{\pi} \equiv \tfrac{1}{2} + \frac{\pi^{**} - \frac{1}{2}}{2\sum_{k=\frac{N+1}{2}}^{N} P(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k) - 1}. \end{aligned} \tag{38}$$

In the Online Appendix, we compare $\hat{\pi}$ and $\tilde{\pi}$ and show that $\hat{\pi} \leq \tilde{\pi} \Leftrightarrow g\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}\right) \leq g\left(\frac{1}{2} + \Lambda\right)$ is satisfied if and only if $\frac{1}{2} + \frac{1}{2\sqrt{N}} \geq \frac{1}{2} + \Lambda \Leftrightarrow \Lambda \leq \frac{1}{2\sqrt{N}}$. Note also that $\Lambda \leq \frac{1}{2\sqrt{N}} \Leftrightarrow \tilde{\pi} \leq 1$, as follows from (38). Hence, if $\Lambda \leq \frac{1}{2\sqrt{N}}$, then $\hat{\pi} \leq \tilde{\pi}$ and $\tilde{\pi} \leq 1$, so the advisor improves the quality of decision-making compared to the benchmark case if and only if $\pi > \tilde{\pi}$. If $\Lambda > \frac{1}{2\sqrt{N}}$, then $\hat{\pi} > \tilde{\pi}$

and $\tilde{\pi} \geq 1$, so the advisor never improves the quality of decision-making. Hence, in both cases, the advisor improves the quality of decision-making compared to the benchmark case if and only if $\pi > \tilde{\pi}$.

It remains to prove the second part of the proposition. Using (17), $\tilde{\pi}$ exceeds one if and only if

$$\frac{1}{2} + \frac{\pi^{**} - \frac{1}{2}}{2\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k\right) - 1} > 1 \Leftrightarrow \pi^{**} > \sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k\right).$$

By definition, $\pi^{**} = \sum_{k=\frac{N+1}{2}}^{N} P(p_0, N, k)$, where $p_0 \equiv pq_0^* + \frac{1-q_0^*}{2} > \frac{1}{2}$. As shown in the proof of Proposition 3, $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) > 0$ for $q > \frac{1}{2}$ and hence this inequality is equivalent to $p_0 > \frac{1}{2} + \frac{1}{2\sqrt{N}}$. Simplifying, we get $(2p-1)\,q_0^* > \frac{1}{\sqrt{N}}$.

Proof of Proposition 6. First, consider $q_s > 0$. Then, the planner's problem is $\max_{q_r, q_s} U(q_r, q_s)$ subject to $V_s(q_r, q_s) \geq c$. This problem is equivalent to problem (29). As shown in the proof of Proposition 3, its solution coincides with the equilibrium of the benchmark case without the proxy advisor, i.e., $(q_r, q_s) = (0, q_0^*)$. Second, consider $q_s = 0$. Then, the planner's problem is $\max_{q_r} U(q_r, 0)$, which is solved by $q_r = 1$. Thus, the solution to the planner's problem is either $(0, q_0^*)$ or $(1, 0)$, whichever leads to a higher $U(q_r, q_s)$. Since the probabilities of the correct decision under $(0, q_0^*)$ and $(1, 0)$ are π^{**} and π, respectively, we obtain the statement of Proposition 6.

Proof of Proposition 7. In the benchmark case, Proposition 1 implies that firm value equals zero: if $N > \bar{N}$, then $q^* = 0$, implying $V_0 = 0$. Consider the model with the advisor. It is sufficient to show that $(q_r, q_s) = (0, 0)$ is not an equilibrium. By contradiction, suppose that it is, and consider the proxy advisor's revenue if he sets fee f_m, given by (15). This fee results in $q_r > 0$ and hence strictly positive revenues of the advisor, implying that $(0, 0)$ is not an equilibrium. Hence, the equilibrium firm value with the advisor is always strictly positive.

Note also that in the limit of $N \to \infty$, the equilibrium fraction of shareholders that buys the advisor's recommendation approaches zero. This is because when $N \to \infty$, $f_m \to 0$ and $\underline{f}_1 \to \frac{c}{2p-1}$. Since $\underline{f} \geq \underline{f}_1$, we have $f_m < \underline{f}$ in the limit of $N \to \infty$, and hence the proxy advisor sets fee f_m, which corresponds to $q_r = \frac{1}{\sqrt{N}}$. Because $\lim_{N\to\infty} \frac{1}{\sqrt{N}} = 0$, firm value converges to zero as well when $N \to \infty$. This argument also implies that if N is above a certain threshold, the equilibrium features complete crowding out of private information acquisition.

Proof of Proposition 8. After the seller has chosen π, the subgame becomes identical to the basic model, so its equilibrium is given by Proposition 4. Denote the expected revenues by the seller for a given choice of π by $R(\pi) = N \max_f f q_r(f, \pi)$, where $q_r(f, \pi)$ is given by Proposition 4. It is worth noting that $R(\pi)$ is strictly increasing in π. The optimal choice of precision, $\pi^*(t)$, solves

$$\pi^*(t) \in \arg\max_{\pi} \{R(\pi) - C(\pi, t)\}.$$

Since $R(\pi)$ is independent from t and $\frac{\partial^2}{\partial\pi\partial t} C(\pi, t) > 0$, function $R(\pi) - C(\pi, t)$ is submodular. Therefore, Topkis's theorem (Topkis, 1978), $\frac{\partial\pi^*(t)}{\partial t} \overset{\text{sign}}{=} -\frac{\partial^2 C(\pi^*(t), t)}{\partial\pi\partial t} < 0$. That is, higher information acquisition cost of the advisor leads to a less precise signal. To see that comparative statics in t maps into comparative statics in π, consider the extreme cases. If $t \to \infty$, then $C(\pi, t)$ is close to infinity for any $\pi > \frac{1}{2}$. Thus, $\lim_{t\to\infty} \pi^*(t) = \frac{1}{2}$. If $t \to 0$, then $\frac{\partial}{\partial\pi} C(\pi, t)$ is close to zero for any $\pi < 1$. Thus, the optimal precision of the signal will be arbitrarily close to 1, provided that $R'(\pi)$ is bounded away from zero at $\pi = 1$. This fact follows from an application of the envelope theorem to $R(\pi)$ and the fact that the partial derivative of (13) in π is bounded away from zero for any $f > 0$ (for $\pi \to 1$, we know that the equilibrium cannot feature incomplete crowding out of private information production, because (21) has no solution for $\pi = 1$). Therefore, $\pi^*(t)$ is decreasing in t, ranging from $\lim_{r\to 0} \pi^*(t) = 1$ to $\lim_{r\to\infty} \pi^*(t) = \frac{1}{2}$.

Define $\tilde{t}$ as $\pi^*\left(\tilde{t}\right) = \tilde{\pi}$, provided that $\tilde{\pi} < 1$, and $\tilde{t} = \infty$, if $\tilde{\pi} \geq 1$. Then, Proposition 5 and the argument in the previous paragraph imply that the equilibrium firm value exceeds the equilibrium firm value in the model without the advisor if and only if $t < \tilde{t}$.

Auxiliary Lemma A1. *Function $f(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$, where $\varphi(x)$ is defined by (30), is strictly decreasing and strictly concave.*

The proof is relegated to the Online Appendix.