Economic Growth through Diversity in Beliefs*

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January 29, 2024

*We would like to thank Yong Chen, Paula Cocoma (discussant), Giuliano Curatola (discussant), Tristan Fitzgerald, Jonathan Halket, Shane Johnson, Hagen Kim, Jim Kolari, Adam Kolasinksi, Arvind Mahajan, Thien Nguyen (discussant), Stavros Panageas (discussant), and Harold H. Zhang (discussant), seminar participants at Indiana University, the University of Iowa, the University of Southern Denmark, Texas A&M University, and conference participants at the 7th SAFE Asset Pricing Workshop, the 2020 meeting of the Northern Finance Association, the 2021 Lone Star Finance Conference, the 2022 meeting of the Midwest Finance Association, the 20th Macro Finance Workshop, and the 2023 meeting of the American Finance Association, 2023 meeting of the European Finance Association, and 2024 meeting of the Econometric Society. The titles of earlier versions of this paper were "Disagreement, Innovations, and Endogenous Growth", "Growth through Experimentation", and "Growth through Diversity".

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Abstract

We study a macro-finance model with entrepreneurs who have different views on identifying the "next big idea," leading them to pursue distinctly different paths. Their different views and consequent engagement in a broad array of strategies and actions serve to tap into the full spectrum of societal ideas, thereby fostering economic growth that would be unattainable without such diversity. The resulting benefits for future generations come at the cost of higher wealth and consumption inequality because a few entrepreneurs will *ex*-post be successful while most entrepreneurs will fail. Venture capital funds and taxes enhance risk sharing among entrepreneurs, stimulating innovation and growth unless high taxes deplete entrepreneurial capital. Redistribution via taxes reduces inequality and can raise interest rates. Nevertheless, a tradeoff exists between risk-sharing and the exertion of costly effort, giving rise to a hump-shaped economic growth curve when plotted against tax rates.

Keywords: Diverse beliefs, Disagreement, Societal Ideas, Entrepreneurship, Externality, Endogenous Growth, Innovations, Tax distortions, and Venture capital.

JEL Classification: D51, G10, G11, G18, L26, O30, O40

1 Introduction

Entrepreneurs, by innovating across a diverse range of ideas, play an important role in driving economic growth. However, innovations are also associated with substantial risk. By their nature, innovations require venturing into the unknown, where it is difficult to predict the outcome. There are numerous examples and anecdotes of successful inventions that were discarded as impossible or even ridiculed but turned out to be hugely successful. Of course, there are also numerous examples of innovations, most of which we do not know about today, that turned out to be spectacular failures. Consider the invention of the light bulb by Thomas Edison in 1879. Not everyone at that time was enthusiastic about the prospects of this innovation. Henry Morton, a renowned scientist and president of the Stevens Institute of Technology, stated in the New York Times on December 28, 1879, that "Everyone acquainted with the subject will recognize it as a conspicuous failure." Henry Morton was not the only skeptic. A British parliamentary committee commented on the light bulb: "... good enough for our transatlantic friends ... but unworthy of the attention of practical or scientific men." While these statements are ridiculous in retrospect, at the time, it was not an uncommon view among intellectuals. Edison was also not the first one to invent an incandescent lamp. Friedel and Israel (2010) discusses more than 20 unsuccessful inventors prior to Edison's version. What made Edison succeed while so many others failed maybe be understood with the benefit of hindsight, but it was clearly important that so many were not discouraged or even prevented from trying because the light bulb is considered one of the most important innovations in history.

The story about the light bulb is far from unique. Many important inventions share similar narratives of uncertainty, doubt, and most notably, widely diverse views about their potential. Predicting, let alone reaching consensus on, the next big idea and the appropriate strategy for its successful implementation, is exceedingly challenging. In a similar vein, entrepreneurs often subject themselves to massive risks, which, when observed by outsiders, seem to offer low average returns. For instance, Moskowitz and Vissing-Jørgensen (2002) demonstrate that the average return to non-publicly traded firms is not higher than that of public firms, even though ownership of these private firms is highly concentrated (often over 70 percent in a single firm). This lack of diversification and the seemingly poor risk-return tradeoff, from an outsider's perspective, is difficult to justify, leading the authors to refer to this phenomenon as the "entrepreneurial puzzle".

We study the role of belief diversity for economic growth. Does belief diversity spur economic growth, or does it divert resources to unproductive ventures? Might it exacerbate wealth inequality? Who benefits, and who bears the cost? Can it provide a new perspective on the entrepreneurial puzzle? This paper addresses these questions using an equilibrium model with diverse views on innovation success. Drawing parallels with the invention of the light bulb—where inventors held very different beliefs–entrepreneurs in our model hold diverse beliefs about the right path forward. If their confidence in success is high, they pursue entrepreneurship. However, ex post, only very few entrepreneurs succeed. Becoming an entrepreneur entails costs and, due to a "skin in the game" constraint, risk cannot be easily diversified.

We demonstrate that belief diversity is crucial for economic growth, as it encourages entrepreneurs to explore a wider array of ideas. This effectively means that society taps into the entire spectrum of innovative possibilities. Moreover, this diversity alleviates the skin in the game constraints arising from ex ante moral hazard issues. Although a majority of these ideas fail ex post, the few successful ones contribute significantly to society and to substantial economic growth for future generations. However, this surge in economic growth coincides with an increase in wealth and consumption inequality. Ex post, entrepreneurs bear the bulk of the cost, while the rate of intergenerational growth among non-entrepreneurs equals aggregate growth in the economy. Furthermore, we show that these entrepreneurial activities yield low expected returns coupled with substantial idiosyncratic risk, thereby shedding light on the entrepreneurial puzzle.

We study an overlapping generations model in the spirit of Blanchard (1985) and Gârleanu and Panageas (2015). Agents entering the economy can choose to become entrepreneurs and innovate. To innovate, an entrepreneur must place a significant portion of her endowment at risk. While numerous paths to innovation exist, only one eventually proves successful ex post. This successful innovation triggers positive spillovers to future generations, thereby fuelling sustained growth. Within our model, the variety of agent types corresponds to the multitude of innovative paths. These agents have divergent views on the optimal course, resulting in a distribution of beliefs about each innovative path. After an agent decides to innovate or note, the inherent uncertainty is resolved, and agents solve a classic dynamic consumption-investment problem with complete markets.

The key friction in our model stems from the fact that agents are unable to diversify across multiple avenues of innovation. Although we do not microfound this friction, we attribute it to a skin in the game constraint, underpinned by the moral hazard at play. This constraint necessitates that the entrepreneur retains a minimum portion of the company's equity, which for the sake of simplicity in our model, is represented by the entire equity stake. Furthermore, this friction rules out the potential for pooling across all projects in equilibrium, thereby preserving the element of idiosyncratic risk. We relax this assumption in Section 5, where we introduce government tax redistribution or venture capital funds as potential avenues for risk sharing.

We show that diversity in beliefs among innovators helps circumvent the skin in the game constraint, as the resulting optimism about an invention may make an investment appear to have a high Sharpe ratio, despite a general understanding of its high failure rate on average. This is reminiscent of the common phenomenon where many believe themselves to be better drivers than the average. More importantly, belief diversity works as a sampling device from the distribution of different ways to implement ideas. If everyone has the same view or an institution would settle on a view, e.g., the consensus or expert view, then only one way of innovating would be implemented. While these views may have a higher probability ex ante it would only lead to innovation if it is ex post correct. In our model, there is a continuum of investors with different ideas and therefore the entire distribution of ideas is tested every period, leading to smooth economic growth for society that is much higher than when only one view is implemented. Therefore, diverse views and the ensuing experimentation benefit society as a whole, despite not being advantageous for the average entrepreneur on an ex-ante and ex-post basis. This dynamic inevitably leads to wealth and consumption inequality.

Our study examines two mechanisms for entrepreneurial risk sharing: taxation and venture capital fund investment. Absent the costly effort to innovate, both mechanisms can mitigate entrepreneurial risk, thereby enhancing the prospects of innovation. This diminished entrepreneurial risk bolsters economic growth, provided the tax framework does not excessively deplete entrepreneurial capital. Tax redistribution consistently alleviates inequality. However, it only influences asset prices when taxes are reallocated across different demographic cohorts. Introducing costly effort to innovate leads to a tradeoff between risk-sharing and exerting effort. As tax rates rise from zero, they contribute to improved risk sharing among entrepreneurs, stimulating greater levels of innovation and, consequently, economic growth. This is because the redistributed wealth, through taxation, provides a safety net that encourages entrepreneurs to take calculated risks. However, if tax rates escalate too high, they may diminish the incentives for entrepreneurs to exert effort and innovate, due to the reduced potential for personal gain. This can lead to a drop in the likelihood of innovation and subsequent economic growth.[1] Hence, the relationship between tax rates, entrepreneurial innovation, and economic growth is not linear but rather hump-shaped.

Our model is deliberately stylized yet highly tractable, yielding all quantities in closed form, thus making the underlying economic forces transparent. For instance, we do not factor in aggregate uncertainty, and we assume agents have logarithmic utility. Any uncertainty related to innovations is resolved immediately. Furthermore, we propose a continuum of agents, thus applying the law of large numbers. These features contribute significantly to the model's tractability. However, they can be relaxed, albeit at the expense of diminishing the transparency of our economic argument.

Our paper is related to the literature examining the asset pricing implications of technological innovation, such as Garleanu, Panageas, and Yu (2012), Kogan, Papanikolaou, and Stoffman (2013), Kung (2015), Kung and Schmid (2015), Garleanu, Panageas, Papanikolaou, and Yu

[1] Akcigit and Stantcheva (2020) provides a survey of the evidence on taxation and innovation. Akcigit, Baslandze, and Stantcheva (2016) document how superstar inventors are highly sensitive to tax rates and that it affects where they locate.

(2016), Andrei and Carlin (2023), Haddad, Ho, and Loualiche (2018), Lin, Palazzo, and Yang (2017), and Opp (2019). We also connect to papers looking at the impact of taxes on asset prices through the growth margin, such as Croce, Kung, Nguyen, and Schmid (2012) and Croce, Nguyen, and Schmid (2012). We also relate to papers studying wealth inequality and asset prices, such as, Gomez (2016), Pástor and Veronesi (2016), and Pastor and Veronesi (2018). We differ from these papers by showing how belief diversity over ideas allows entrepreneurs to try different innovation paths in equilibrium, which is important for long-run economic growth. Experimenting with a wider spectrum of ideas increases the likelihood that entrepreneurs as a whole find breakthrough innovation that increases aggregate trend growth.

Our paper also relates to the literature on that studies how heterogeneous beliefs effects asset prices such as Miller (1977), Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), Basak (2000), Scheinkman and Xiong (2003), Basak (2005), Berrada (2006), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic, Jouini, Malamud, and Napp (2012), Simsek (2013b), Simsek (2013a), Bhamra and Uppal (2014), Buraschi, Trojani, and Vedolin (2014), Chien, Cole, and Lustig (2016), Cujean and Hasler (2017), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018), Collin-Dufresne, Johannes, and Lochstoer (2017), Ehling, Graniero, and Heyerdahl-Larsen (2018), Borovička (2019), and Heyerdahl-Larsen and Illeditsch (2023). Our paper makes a unique contribution to this literature by introducing a heterogeneous beliefs model that highlights the crucial role of diverse views in promoting innovation. Additionally, we analyze the implications of diverse views for interst rates and consumption/wealth inequality, which, to the best of our knowledge, has not been explored in previous studies.

Finally our paper also relates to the literature that studies the asset pricing implication of overlapping generations (OLG) models, such as Constantinides, Donaldson, and Mehra (2002), Gomes and Michaelides (2005), Acharya and Pedersen (2005), Gârleanu, Kogan, and Panageas (2012), Gârleanu and Panageas (2015), Gârleanu and Panageas (2021) Kogan, Papanikolaou, and Stoffman (2019), Ehling, Graniero, and Heyerdahl-Larsen (2018), Heyerdahl-Larsen and

Illeditsch (2021), and Heyerdahl-Larsen and Illeditsch (2023). Our paper extends the literature on OLG models by employing this tractable framework to study endogenous growth. While previous studies have demonstrated the usefulness of OLG models in analyzing heterogeneity and obtaining a non-degenerate stationary wealth distribution, we uniquely incorporate an innovation stage that introduces dynamic aspects to the model. By doing so, we provide insights into the endogenous growth process within this framework, shedding light on the interplay between heterogenous beliefs, innovation, wealth distribution, and asset pricing.

2 Exogenous growth

In this section, we introduce the benchmark model with exogenous output growth. There is no heterogeneity within birth cohorts but the exogenous growth will lead to heterogeneity across birth cohorts. In the next section, we introduce a decision to become an entrepreneur when entering the economy. An agent that chooses to be an entrepreneur will either be successful or fail with their innovation decision. Successful innovation will generate positive output externalities for future generations, implying endogenous growth and heterogeneity within and across birth cohorts. Both models are based on a continuous-time overlapping generations setting in the spirit of Blanchard (1985) and, more recently, Gârleanu and Panageas (2015).

Every agent in the economy faces a stochastic time of death T that is exponentially distributed with the hazard rate, $\nu > 0$. A new cohort of mass ν is born every period. Consequently, the population size remains constant:

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)}\, ds = 1, \tag{1}$$

where $\nu e^{-\nu(t-s)}$ denotes the population density. An agent born at time s is entitled to the endowment stream $Y_{s,t}$ as long as the agent is alive. Enforcing the aggregate resource constraint by integrating over all agents currently alive leads to

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t}\, ds = Y_t, \tag{2}$$

where Y_t denotes total output at time t. There is no aggregate uncertainty in the economy and the output growth rate μ_Y is exogenous. Hence, the dynamics of Y_t are

$$dY_t = \mu_Y Y_t\, dt. \tag{3}$$

Let $Y_{s,t} = \gamma Y_s$ with $\gamma > 0$, and thus there is no endowment growth within a birth cohort. If there is aggregate growth, new birth cohorts receive more output, and γ exceeds one. Specifically, differentiating the aggregate resource constraint given in Equation (2) w.r.t. time t (by applying the integral rule of Leibniz) leads to

$$\mu_Y = \nu(\gamma - 1). \tag{4}$$

Hence, aggregate output growth depends on the endowment growth of the new cohort, $\gamma - 1$, and the probability, νdt, of having a new cohort over the next instant.

Agents can trade two assets: (i) an instantaneously risk-free asset in zero-net-supply and (ii) a life insurance/annuity contract that is offered by a competitive insurance industry as in Blanchard (1985) and Gârleanu and Panageas (2015). The dynamics of the real risk free asset with price B_t are

$$dB_t = rB_t\, dt, \tag{5}$$

where the real short rate, r, is determined in equilibrium. The life insurance/annuity contract pays the actuarily fair rate ν per unit of wealth in the case of an annuity and it charges the rate ν per unit of wealth in the case of a life insurance. Hence, the dynamics of the insurance contract from the agent's perspective are

$$d\mathcal{L}_t = \nu W_t^{\mathcal{L}}\, dt, \qquad \mathcal{L}_T = -W_T^{\mathcal{L}}, \qquad \forall t \leq T, \tag{6}$$

where $\mathcal{L}_t$ denotes the value of the insurance contract at time t and $W_t^{\mathcal{L}}$ is the amount of wealth invested in the insurance contract. It is optimal for an agent with positive financial wealth to annuitize all her wealth because she does not have any bequest motive and, hence, does not get any utility from dying with positive financial wealth. Moreover, agents have to buy life insurance for their negative financial wealth to avoid default at death because they are no longer entitled to an income stream after death. The financial wealth of all agents currently alive is always zero, and thus the insurance market clears.

Agents have log-utility and thus an agent born at time s has life time utility,

$$U_s = E_s\left[\int_s^T e^{-\rho(t-s)} \log C_{s,t}\, dt\right] = \int_s^\infty e^{-(\rho+\nu)(t-s)} \log C_{s,t}\, dt, \tag{7}$$

where $C_{s,t}$ denotes time-t consumption of an agent born at time s. Mortality risk, captured by the random time of death T, increases the effective time discount rate from ρ to $\rho+\nu$. Once an agent is born, she can trade the risk free asset and the insurance contract, and therefore faces a dynamically complete market. Hence, we can solve the model by maximizing expected utility given in Equation (7) subject to the static budget condition (see Cox and Huang (1989) and Karatzas and Shreve (1998)),

$$E_s\left[\int_s^T \frac{M_t}{M_s} C_{s,t}\, dt\right] = \int_s^T \frac{M_t}{M_s} Y_{s,t}\, dt \equiv H_s, \tag{8}$$

where M_t is the discount factor with dynamics $dM/M = -r\, dt$ since there is no aggregate uncertainty. The first order conditions (FOC) is

$$e^{-\rho(t-s)} \frac{1}{C_{s,t}} = \kappa_s \frac{M_t}{M_s} = \kappa_s e^{-r(t-s)}, \tag{9}$$

where κ_s denotes the Lagrange multiplier of the static budget constraint of birth cohort s. Solving for $C_{s,t}$ leads to the optimal path for consumption

$$C_{s,t} = \kappa_s^{-1} e^{-\rho(t-s)} \frac{M_s}{M_t} = C_{s,s} e^{(r-\rho)(t-s)}, \qquad \forall\, s \le t \le T. \tag{10}$$

Total wealth at time t of an agent born at time s denoted by $W_{s,t}$ is the sum of her financial wealth $\hat{W}_{s,t}$ and her endowment value $H_{s,t}$. Initially, at the time of birth, an agent's financial wealth is zero. Subsequently, it may increase or decrease, reflecting gains or losses resulting from trading activities. Every agent of birth cohort s is entitled to the constant endowment stream $Y_{s,t} = \gamma Y_s$ and thus the value of her endowment stream at time t is

$$H_{s,t} = E_t \left[\int_t^T \frac{M_u}{M_t} Y_{s,u} \, du \right] = \gamma Y_s \psi_t, \qquad \forall\, s \le t \le T, \tag{11}$$

where ψ_t is the price of a life annuity that continuously pays one unit of the consumption good until time of death T. Specifically,

$$\psi_t = E_t \left[\int_t^T \frac{M_u}{M_t} \, du \right] = \int_t^\infty e^{-(r+\nu)(u-t)} \, du = \frac{1}{\nu + r} \equiv \psi. \tag{12}$$

At every point in time t, an agent's total wealth has to finance her consumption stream and thus

$$W_{s,t} = \hat{W}_{s,t} + H_{s,t} = E_t \left[\int_t^T \frac{M_u}{M_t} C_{s,u} \, du \right] = \frac{1}{\nu + \rho} C_{s,t}, \qquad \forall\, s \le t \le T. \tag{13}$$

The last equality follows from inserting optimal consumption given in Equation (10) for $C_{s,u}$. This verifies the well-established results that agents with log-utility consume at a constant rate equal to their effective time discount rate out of their wealth.

To pin down optimal consumption in Equation (10), we still need to solve for initial consumption $C_{s,s}$ or, equivalently, the initial consumption per unit of output defined as $\beta_s = C_{s,s}/Y_s$. To determine β_s we value the endowment stream of an agent at birth and then use the fact that agents consume a constant fraction out of their wealth (see Equation (13)).

Agents are born with no financial wealth and receive a constant endowment stream, and thus,

$$\beta_s = \frac{C_{s,s}}{Y_s} = (\nu+\rho)\frac{W_{s,s}}{Y_s} = (\nu+\rho)\frac{H_s}{Y_s} = (\nu+\rho)\gamma\psi \equiv \beta. \tag{14}$$

To determine the risk-free rate, we combine the FOC for optimal consumption with the aggregate resource constraint and get

$$M_t Y_t = \int_{-\infty}^{t} \nu e^{-(\nu+\rho)(t-s)} \beta Y_s M_s \, ds. \tag{15}$$

Differentiating both sides of the previous equation w.r.t. time t leads to the risk-free rate

$$r = \rho + \mu_Y + \nu(1-\beta). \tag{16}$$

The constants $r = \rho$ and $\beta = \gamma$ satisfy Equations (14) and (16). Hence, the equilibrium interest rate is constant and does not depend on output growth. Intuitively, the interest rate is determined by all agents currently alive, and even though there is endowment growth from generation to generation when γ exceeds one, nobody experiences any endowment growth after birth, and therefore the interest rate does not depend on growth.

To determine the wealth-output ratio we compute total wealth in the economy by integrating over the total wealth of all agents currently alive. Specifically,

$$\frac{W_t}{Y_t} = \frac{1}{Y_t}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} W_{s,t} \, ds = \frac{1}{\nu+\rho} = \psi. \tag{17}$$

The market portfolio is a claim on the total wealth of all agents currently alive, and thus the price-dividend ratio is $\phi \equiv P_t/Y_t = \psi$. We summarize the results in Proposition 1.

Proposition 1. *In the exogenous growth model with endowment heterogeneity across birth cohorts, there is an equilibrium in which all agents consume their endowment, that is,*

$$C_{s,t} = C_{s,s} = \gamma Y_s = Y_{s,t}, \qquad \forall\, s \le t \le T. \tag{18}$$

The risk-free rate is constant and equal to the subjective time discount rate, that is, $r = \rho$. *The price of a life annuity is* $\psi = \frac{1}{\nu+\rho}$ *and the price-dividend ratio is equal to* $\phi = \frac{1}{\nu+\rho}$.

There is no consumption or wealth inequality within a birth cohort, and the annualized log consumption growth rate across birth cohorts is equal to aggregate output growth. Specifically,

$$\frac{1}{\Delta}\log\left(\frac{C_{s+\Delta,t}}{C_{s,t}}\right) = \mu_Y, \qquad \forall\, \Delta > 0 \qquad \textit{and} \qquad s \leq t \leq T. \tag{19}$$

The risk-free rate does not depend on output growth and thus there is no growth in individual consumption which usually occurs when the risk-free rate exceeds the subjective time discount factor. However, the price-dividend ratio does not depend on growth which is surprising since the economy is growing and the risk-fee rate does not depend on growth. The reason is that we define the market portfolio as a claim on total wealth of all agents currently alive which does not include wealth that is generated by future generations.

We conclude this section by contrasting our findings with an exogenous growth model without output heterogeneity, as detailed in Proposition 2. In this model, every agent's consumption increases at the same rate as the aggregate output, consequently causing the risk-free rate to also rely on growth. Given log-utility, cash flows grow at the same pace as the rate at which they are discounted, meaning the price-dividend ratio remains independent of growth.

Proposition 2. *In the exogenous growth model without endowment heterogeneity, that is,* $Y_{s,t} = Y_t$ *there is an equilibrium in which all agents consume their endowment, that is,*

$$C_{s,t} = Y_s e^{(r-\rho)(t-s)} = Y_t, \qquad \forall\, s \leq t \leq T. \tag{20}$$

The risk-free rate is constant and equal to $r = \rho + \mu_Y$, *the price of a life annuity is* $\psi = \frac{1}{\nu+r}$, *and the price-dividend ratio is* $\phi = \frac{1}{\nu+\rho}$.

3 Endogenous growth

In this section, we introduce the choice to become an entrepreneur, and this choice will feed back into aggregate economic growth. Specifically, we consider the same demographic structure as in Section 2 in which each agent receives the endowment stream $Y_{s,t} = \gamma Y_s$. The choice to become an entrepreneur or innovator and the ex-post separation of agents in successful, unsuccessful, and non-innovators endogenously determines γ and hence leads to endogenous growth and inequality within and across birth cohorts. Successful innovators enjoy the reward of a larger endowment, generating inequality within a cohort. The successful innovator passes on the benefits to future generations through positive output spillovers shared by future cohorts as they get to start off with a higher level of the endowment. While we do not explicitly model the aggregate production technology, we can interpret the endowment spillovers as intertemporal knowledge externalities from successful innovation that raise the productive efficiency for future generations (e.g., Aghion and Howitt (1992) and Grossman and Helpman (1993)).

3.1 Innovation

Agents can choose to become entrepreneurs and engage in an activity that may lead to an innovation. Only entrepreneurs can innovate. Specifically, an agent entering the economy at time s has an endowment stream Y_s. They can only make the decision to become an entrepreneur at the time of entry. If the agent chooses not to innovate, then $Y_{s,t} = Y_s$ for all $s \leq t \leq T$. Hence, any agent who chooses not to innovate will not experience any growth in her endowment stream. An agent that chooses to become an entrepreneur must pay the cost $(1-\delta)Y_s$ with $0 < \delta < 1$. This cost can be thought of as an irreversible investment into the entrepreneurial activity. The entrepreneur must bear the entire investment risk due to an unmodeled skin-in-the-game constraint.[2] Importantly, the choice to become an entrepreneur is made just prior to entering the economy, and all uncertainty about the success of the innovation

[2] Relaxing this assumption by allowing the entrepreneur to sell part of the firm to an outside investor would lead to qualitatively similar results.

is resolved immediately after the decision to innovate has been made.

If the entrepreneur is successful, then her endowment is $Y_{s,t} = A\delta Y_s$ for all $s \leq t \leq T$. Let $A\delta > 1$ because otherwise, an innovation would not raise output even if successful. If the entrepreneur is unsuccessful, then her endowment is $Y_{s,t} = \delta Y_s$ for all $s \leq t \leq T$. The agent's endowment stream is passed on to a new generation when the agent dies. Hence, neither the knowledge created by a successful innovation nor the cost of an unsuccessful innovation vanishes. Hence, we can write the endowment in the three different scenarios as $Y_{s,t} = \omega_i Y_s$ for all $i \in \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$ and with

$$\omega_i = \begin{cases} A\delta > 1, & \text{if } i = \text{se}, \quad \text{that is, the agent is a successful entrepreneur} \\ \delta < 1, & \text{if } i = \text{ue}, \quad \text{that is, the agent is an unsuccessful entrepreneur} \\ 1, & \text{if } i = \text{ne}, \quad \text{that is, the agent is not an entrepreneur.} \end{cases} \tag{21}$$

We focus on a winner-takes-all-innovation economy in which many entrepreneurs try but only a handful capture a large share of the reward. Hence, every period there are $H \gg 1$ possible innovation strategies and only one of them leads to success with a large reward $A\delta \gg 1$. When each of the H strategies is equally likely to succeed, then the expected per period endowment is given by

$$E^0\left[Y_{s,t}\right] = \left(A\frac{1}{H} + \left(1 - \frac{1}{H}\right)\right)\delta Y_s = \left(1 + \frac{1}{H}(A-1)\right)\delta Y_s. \tag{22}$$

Hence, a risk-neutral agent would become an entrepreneur as long as this expectation exceeds the endowment of agents who do not become entrepreneurs (Y_s), that is, she becomes an entrepreneur if $\delta > \delta^* \equiv \frac{H}{H+A-1}$. However, a risk-averse agent might require a large entrepreneurial risk premium due to the skin-in-the-game constraint that prevents her from diversifying and thus, she has to bear the idiosyncratic entrepreneurial risk. Specifically, an agent with log-utility would innovate if

$$E^0\left[\log Y_{s,t}\right] = \log\left(A\delta Y_s\right)\frac{1}{H} + \log\left(\delta Y_s\right)\left(1 - \frac{1}{H}\right) > \log(Y_s), \tag{23}$$

and thus, she innovates if

$$\delta > \delta^{**} \equiv A^{-\frac{1}{H}} > \delta^*. \tag{24}$$

To stress the winner-takes-all innovation economy we consider the following numerical example for the remainder of this paper.

Baseline example. *Suppose there are $H = 1000$ different innovation strategies or projects, and the reward of a successful innovation is very large, that is, $A = 1501$. Hence, $\delta^* = 40\%$ and $\delta^{**} = 99.27\%$.*

A risk-neutral entrepreneur will pursue innovation as long as potential losses do not exceed 60% of her investment should the project fail. However, the inability to diversify entrepreneurial risk—owing to the skin-in-the-game constraint—significantly increases this threshold from 40% to 99.27% with moderate risk aversion. Consequently, this leads to a significantly less innovation except in the rare situations where the cost to innovate is less than 0.73%.

Moreover, when agents are indifferent ex-ante among the H different innovation strategies, the ex-post fraction of successful innovators becomes indeterminate since we do not know the fraction of entrepreneurs who will choose the single successful innovation strategy out of the H possible ones. However, we show in the next section that diverse views about the success of different innovation strategies leads to more innovation because it alleviates the skin-in-the-game constraint and it uniquely determines the ex-post fraction of successful innovators, which is increasing in the degree of diversity. It is crucial to note, however, that the consensus view—defined in the next section as the cross-sectional average across agents beliefs—still maintains that the chance of successful innovation from employing any single strategy remains at $1/H$. Moreover, we will demonstrate in the following sections that the increase in innovation, fostered by diverse views, can have effects on output, potentially raising or even lowering it. Hence, we introduce a benchmark that outlines the social optimum in terms of cost to innovating.

First best benchmark. *If $\delta \geq \delta^* \equiv \frac{H}{H+A-1}$, then it is socially optimal to employ all H strategies and thus eliminate all the entrepreneurial idiosyncratic risk. If $\delta < \delta^*$, then it is* *socially optimal to not employ any of the H strategies. In our baseline example it is socially optimal to innovate if $\delta \geq \delta^* = 40\%$ and forego innovation if $\delta < \delta^* = 40\%$.*

If it is socially optimal to innovate, then entrepreneurs could achieve first best by pooling their endowments and employ all H strategies. We discuss in Section 5 two different mechanisms to share entrepreneurial risk: taxes and a venture capital fund.

3.2 Diverse Views

We define in this section the beliefs of entrepreneurs. These beliefs consist of two elements. The first element reflects a strong preference for a particular innovation strategy that is often shaped by factors such as their visionary goals, unique insights, or specific skill sets and resources. The second element pertains to their level of optimism and confidence in the success of the chosen strategy, which varies among entrepreneurs.

Let p_h denote the subjective probability of an entrepreneur that innovation strategy h leads to success with $p_h > \frac{1}{H}$ and p_j denote the subjective probability that innovation strategy j leads to success with $p_j \equiv \frac{1-p_h}{H-1} < \frac{1}{H}$ for all $j \in \{1, \ldots H\} \backslash \{h\}$. Hence, an entrepreneur who has a strong preference for innovation strategy h behaves as if this strategies is the one out of the H possible ones that most likely succeeds. Moreover, all strategies other than her preferred innovation strategy h are perceived to be equally likely by this entrepreneur. Since only one out of the H possible strategies leads to success, we have that the probability of success and failure is p_h and $1 - p_h$, respectively.

Entrepreneurs not only have different views about the best innovation strategy, but they also differ in their degree of optimism and confidence about their chosen innovation strategy. Hence, we assume that there is a unit interval of agents who differ across two dimensions: (i) their preferred innovation strategy indexed by h and (ii) their perceived probability of success p_h. For each of the H different innovation strategies, there is an equal mass of agents with different beliefs about the success probability, that is, $p_h = \frac{\Delta_h}{H}$ for all $h \in \{1, \ldots, H\}$. The parameter Δ_h, which captures the degree of optimism and confidence across entrepreneurs who chose innovation strategy h, is uniformly distributed on the interval $\left[1, \bar{\Delta}\right]$ with $1 \leq \bar{\Delta} < H$. The lower bound of 1 for Δ_h guarantees that for each innovation strategy h the mass $\frac{1}{H}$ of entrepreneurs thinks that strategy h most likely succeeds and the upper bound of $\bar{\Delta} < H$ guarantees that no entrepreneur thinks that success is certain. Moreover, an entrepreneur who is optimistic about a specific innovation strategy is pessimistic about all others. By symmetry, we have that the consensus view of all entrepreneurs about the probability that a specific innovation strategy leads to success is $1/H$. However, the consensus probability of success for entrepreneurs who prefer innovation strategy h is

$$\bar{p}_{success} \equiv \int_1^{\bar{\Delta}} \frac{\Delta_h}{H} \frac{1}{\bar{\Delta} - 1} d\Delta_h = \frac{1 + \bar{\Delta}}{2H}. \tag{25}$$

The consensus view of all entrepreneurs about the success probability is also equal to $\bar{p}_{success}$ because the distribution of beliefs is the same for all innovation strategies. Similarly, the belief dispersion across all entrepreneurs defined as the cross-sectional standard deviation of beliefs is $\bar{\Delta}_{success} = \frac{\bar{\Delta}-1}{H\sqrt{12}}$ since

$$\bar{\Delta}^2_{success} \equiv \int_1^{\bar{\Delta}} \left(\frac{\Delta_h}{H} - \bar{p}_{success} \right)^2 \frac{1}{\bar{\Delta} - 1} d\Delta_h = \frac{1}{12} \left(\frac{\bar{\Delta} - 1}{H} \right)^2. \tag{26}$$

We measure diversity across investors with the diversity index $\mathcal{D} = \bar{\Delta} - 1$, which monotonically increases the consensus success probability and belief dispersion but holds the consensus probability that a specific innovation strategy leads to success equal to $1/H$.

We now discuss the ex-ante decision to become an entrepreneur. This decision is the same for each innovation strategy, and thus we drop the subscript h and use the parameter Δ when referring to an agent's perceived success probability $p = \Delta/H$. The expected per period endowment growth from innovating as perceived by an agent with belief Δ is[3]

$$E^{\Delta}\left[\frac{Y_{s,t}}{Y_s}\right] = \left(A\frac{\Delta}{H} + \left(1 - \frac{\Delta}{H}\right)\right)\delta = \left(1 + \frac{\Delta}{H}(A-1)\right)\delta. \tag{27}$$

This expectation is strictly increasing in the degree of optimism about the success probability of the entrepreneur's preferred innovation strategy measured given by Δ/H and attains its minimum and maximum if $\Delta = 1$ and $\Delta = \bar{\Delta}$, respectively. However, the risk is also changing with Δ because the variance of the per period endowment growth from innovating is

$$Var^{\Delta}\left(\frac{Y_{s,t}}{Y_s}\right) = (A-1)^2 \frac{\Delta}{H}\left(1 - \frac{\Delta}{H}\right)\delta^2. \tag{28}$$

The risk is a quadratic function of Δ that is maximized when $\Delta = \frac{H}{2}$.

To illustrate the entrepreneurial choice model we set $\bar{\Delta} = 199$ in our baseline example.

Baseline example. *Suppose there are* $H = 1000$ *different innovation strategies or projects, and the reward of a successful innovation is very large, that is,* $A = 1501$. *Hence,* $\delta^* = 40\%$ *and* $\delta^{**} = 99.27\%$. *Let,* $\bar{\Delta} = 199$ *and thus the consensus success probability and belief dispersion is* $\bar{p}_{success} = \frac{1+\bar{\Delta}}{2H} = 10\%$ *and* $\bar{\Delta}_{success} = \frac{\bar{\Delta}-1}{H\sqrt{12}} = 5.72\%$, *respectively. Moreover, the maximum success probability is less than* 20%.

We now discuss the ex post outcome. The belief distribution of entrepreneurs is the same for each innovation strategy and thus, if a fraction α decides to choose innovation strategy h given their beliefs, then the same fraction chooses innovation strategy j for all $j \in \{1, \ldots H\}\backslash\{h\}$. Hence, by the law of large numbers we have with probability one that $\frac{\alpha}{H}$ entrepreneurs will be successful, $\alpha\left(1 - \frac{1}{H}\right)$ will be unsuccessful, and $(1-\alpha)$ will not become entrepreneurs.

[3]The qualitative results in this Section do not change if we assume that agents need to exert effort when innovating because an agent who innovates has to bear all the entrepreneurial risk. We relax this assumption in Section 5 when we allow entrepreneurs to share their risk.

3.3 Decision to innovate

Agents have log-utility and a time discount rate denoted by ρ. They differ according to their innovation strategy, indexed by h, and their perceived probability of success, indexed by Δ. We introduce the function $\iota(\cdot)$ to classify every agent of type (h, Δ) within a birth cohort s into one of three categories: successful entrepreneur (se), unsuccessful entrepreneur (ue), or an agent that does not innovate (ne). Specifically, $\iota: \{1, \ldots, H\} \times \left[1, \bar{\Delta}\right] \rightarrow \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$. We solve the decision problem to innovate by backward induction. The expected utility of a type (h, Δ) agent when entering the economy, after the uncertainty about the innovation decisions is revealed, is

$$U_s^i = E_s\left[\int_s^T e^{-\rho(t-s)} \log C_{s,t}^i \, dt\right] = \int_s^\infty e^{-(\rho+\nu)(t-s)} \log C_{s,t}^i \, dt, \qquad \forall i = \iota(h, \Delta) \in \mathcal{I}, \tag{29}$$

where T and ν denote the uncertain time of death and hazard rate as in Section 2, respectively. Agent i faces dynamically complete markets once the innovation uncertainty has been resolved and thus maximizes expected utility given in Equation (29) subject to her static budget constraint

$$W_{s,s}^i = E_s\left[\int_s^T \frac{M_t}{M_s} Y_{s,t}^i \, dt\right] = \int_s^\infty e^{-(\nu+r)(t-s)} Y_{s,t}^i \, dt = \omega_i Y_s \psi, \tag{30}$$

where $M_t/M_s = e^{-r(t-s)}$ denotes the discount factor as in Section 2. The valuation ratio is equal to the price of a life annuity, ψ, given in Equation (12) because the endowment stream, while different across the three types, remains constant over time. The level of the endowment stream for the different types is represented by ω_i which is given in Equation (21). Optimal consumption at time t of agent i born at time s is

$$C_{s,t}^i = C_{s,s}^i e^{-\rho(t-s)} \frac{M_s}{M_t} = (\rho + \nu) W_{s,s}^i e^{(r-\rho)(t-s)} \qquad \forall\, s \leq t \leq T. \tag{31}$$

Plugging Equation (31) into lifetime expected utility given in equation (29) leads to

$$U_s^i = \frac{\log\left(W_{s,s}^i\right)}{\rho+\nu} + \frac{log\,(\rho+\nu)}{\rho+\nu} + \int_s^\infty e^{-(\rho+\nu)(t-s)} \log\left(\frac{M_s}{M_t}\right)\, dt. \tag{32}$$

Substituting in for the wealth of agent i born at time s given in equation (30) implies

$$U_s^i = \frac{\log\left(\omega^i\right)}{\rho+\nu} + \underbrace{\frac{\log\left(\rho+\nu\right)+\log(\psi_s Y_s)}{\rho+\nu} + \int_s^\infty e^{-(\rho+\nu)(t-s)} \log\left(\frac{M_s}{M_t}\right)\, dt,}_{=\bar{U}_s} \tag{33}$$

where the second term of lifetime utility is independent of the agent's type. Hence, an agent born at time s will choose to become an entrepreneur if the expected lifetime utility from innovating exceeds the lifetime expected utility from not innovating. Specifically,

$$EU_s^e = \frac{\Delta}{H}\frac{\log\left(A\delta\right)}{\rho+\nu} + \left(1-\frac{\Delta}{H}\right)\frac{\log\left(\delta\right)}{\rho+\nu} + \bar{U}_s > \frac{\log\left(1\right)}{\rho+\nu} + \bar{U}_s = EU_s^{ne}. \tag{34}$$

Let Δ^*/H denote the success probability for which agents are indifferent between innovating and not innovating. Hence, agents decide to innovate if their success probability exceeds the threshold Δ^*/H with

$$\Delta^* = -H\frac{\log\ \delta}{\log A} \geq 0.$$

Neither the valuation ratio nor the discount factor affects the decision to innovate and hence the threshold Δ^* is constant. To determine the equilibrium fraction of entrepreneurs for each cohort, we determine the probability that belief type Δ exceeds threshold Δ^* using the cross-sectional distribution of belief types Δ, that is, Δ is uniformly distributed on the interval $[1, \bar{\Delta}]$. The threshold is constant, and the cross-sectional beliefs distribution does not vary over time; thus, the equilibrium fraction of entrepreneurs is the same for each birth cohort s. We summarize the results in the next proposition.

Proposition 3. *In equilibrium an agent with type* (h, Δ) *decides to innovate by choosing* *strategy $h \in \{1, \dots, H\}$ if her success probability Δ/H exceeds the threshold Δ^*/H with*

$$\Delta^* = -H \frac{\log\ \delta}{\log A}. \tag{35}$$

The equilibrium fraction of innovators for every birth cohort is

$$\alpha = \mathbb{P}\left(\Delta > \Delta^*\right) = \begin{cases} 1 & \textit{if} \quad \Delta^* \leq 1 \\ \frac{\bar{\Delta} + H \frac{log(\delta)}{log(A)}}{\bar{\Delta} - 1} & \textit{if} \quad \Delta^* \in \left(1, \bar{\Delta}\right) \\ 0 & \textit{if} \quad \Delta^* \geq \bar{\Delta}. \end{cases} \tag{36}$$

The fraction of entrepreneurs is weakly increasing in diversity in beliefs $\mathcal{D}$ because it is strictly increasing in $\mathcal{D}$ for all $\Delta^* \in \left(1, \bar{\Delta}\right)$, that is

$$\frac{\partial \alpha}{\partial \mathcal{D}} = \frac{1}{\mathcal{D}} \left(1 - \alpha\right) > 0 \textit{ for } \Delta^* \in \left(1, \bar{\Delta}\right). \tag{37}$$

Ex-post only one of the H innovation strategies leads to success and thus for birth cohort s the fraction α/H is successful and has total wealth $A\delta\psi Y_s$, the fraction $\alpha(1 - 1/H)$ is unsuccessful and has total wealth $\delta\psi Y_s$, and the fraction $(1 - \alpha)$ does not innovate and has total wealth ψY_s.

The left plot in Figure 1 illustrates the relationship between the equilibrium fraction of entrepreneurs and the parameter δ. Initially, when the cost parameter δ is below $\delta^{***} = A^{-\frac{\bar{\Delta}}{H}} =$ 23%, no innovation takes place since the perceived success probability falls short of the minimum threshold of 20%. However, when the cost of innovating decreases and δ surpasses 23%, the cutoff probability falls below 20%, and the most optimistic agents start to innovate. The red chain-dotted line shows that there is innovation even though it is not socially optimal when δ ranges between 23% and $\delta^* = 40\%$. In contrast, if δ exceeds 40%, as indicated by the blue dashed line, a substantial fraction of agents innovate in equilibrium, even if δ is lower than $\delta^{**} = 99.27\%$. This finding is significant since, without belief diversity, no innovation would occur within this range. We explore the consequences of socially inefficient innovation, which leads to a marginal reduction in output, compared to the positive economic growth resulting from socially efficient innovations in the next subsection.

The right plot in Figure 1 shows that belief diversity ($\mathcal{D}$) significantly increases the likelihood of innovation which can have significant benefits for society, as indicated by the moderate cost scenarios represented by the black and red chain-dotted lines. However, when the cost to innovate becomes very high, as depicted by the blue dashed and purple circled lines, and $\mathcal{D}$ is sufficiently large, the innovations generated are not beneficial for society.

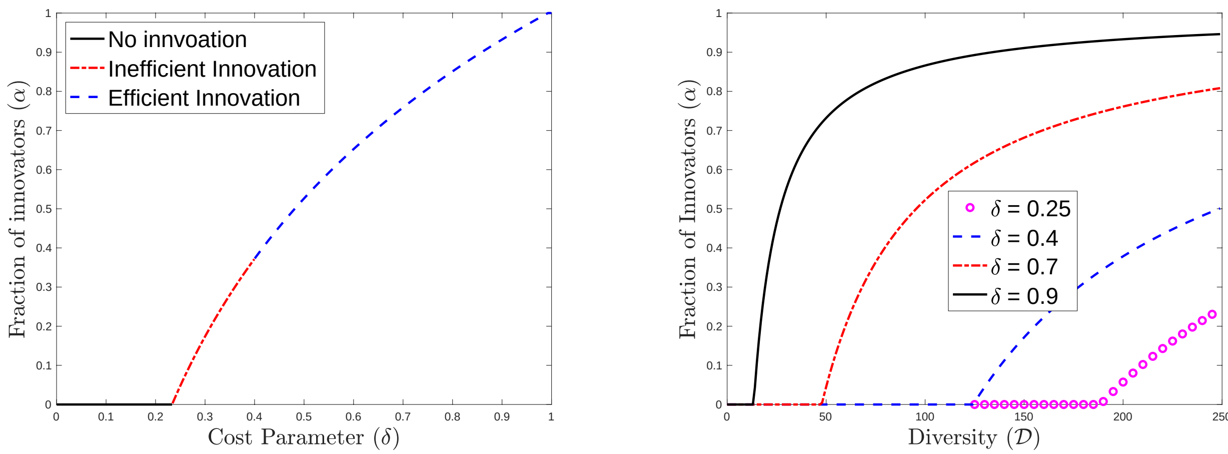


Figure 1: **Equilibrium fraction of innovators (α).** The left plot shows the equilibrium fraction of innovators (α) as a function of the cost parameter (δ) when $\bar{\Delta} = 199$. The black line segment denotes the case when the fraction of innovators is zero, and the red chain-dotted line denotes the segment when the fraction of innovators is positive, but as a group, they should not innovate as they destroy value. The blue dashed line segment shows the case when the fraction of innovators is positive, and they create value as a group. The right plot shows the equilibrium fraction of innovators (α) as a function of belief diversity ($\mathcal{D}$). The black and red-dashed line represents innovation creating value, the purple circled line signifies innovation destroying value, while the blue dashed lines depict innovations that neither add nor destroy value. In the figure, we set $H = 1000$ and $A = 1501$.

We conclude this subsection by setting $\delta = 60\%$ in our baseline example to highlight that there is a lot of socially beneficial innovation with belief diversity.

Baseline example. *Suppose there are $H = 1000$ different innovation strategies or projects, and the reward of a successful innovation is very large, that is, $A = 1501$. Hence, $\delta^* = 40\%$ and $\delta^{**} = 99.27\%$. Let, $\bar{\Delta} = 199$ and thus the consensus success probability and belief dispersion is $\bar{p}_{success} = \frac{1+\bar{\Delta}}{2H} = 10\%$ and $\bar{\Delta}_{success} = \frac{\bar{\Delta}-1}{H\sqrt{12}} = 5.72\%$, respectively. There is no innovation if $\delta < \delta^{***} = 23\%$. Suppose the cost of innovating is 40%, that is, $\delta = 60\%$. In this case the perceived success probability has to exceed $\Delta^*/H = 6.98\%$ in order for an agent to become an* *entrepreneur, the equilibrium fraction of entrepreneurs is* $\alpha = 65.23\%$*, and the ex post fraction of successful and unsuccessful entrepreneurs is* 0.065% *and* 65.17%*, respectively.*

3.4 Economic growth and belief diversity

We derive aggregate output growth and discuss the effects of belief diversity on economic growth in this section. In equilibrium the endowment of all agents currently alive has to sum up to aggregate output. Specifically,

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t}\, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\sum_{i \in \mathcal{I}} \lambda_s^i Y_{s,t}^i \right) ds = Y_t, \tag{38}$$

where we define λ_t^i to be the measure of agents of type $i = \iota(h, \Delta) \in \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$ with $(h, \Delta) \in \{1, \ldots, H\} \times [1, \bar{\Delta}]$ and $\sum_{i \in \mathcal{I}} \lambda_s^i = 1$. Specifically, there is a fraction α/H of successful innovators with endowment stream $A\delta Y_s$, a fraction $\alpha(1 - 1/H)$ of unsuccessful innovators with endowment stream δY_s, and a fraction $(1 - \alpha)$ of non-innovators with endowment stream Y_s. Hence, we have that $Y_{s,t}^i = \omega^i Y_s$ and

$$\lambda_s^i = \begin{cases} \alpha/H & \text{if} \quad i = \text{se} \\ \alpha(1 - 1/H) & \text{if} \quad i = \text{ue} \\ (1 - \alpha) & \text{if} \quad i = \text{ne}. \end{cases} \tag{39}$$

It follows that the aggregate endowment stream of birth cohort s is $Y_{s,t} = \gamma Y_s$ with

$$\gamma = \frac{1}{Y_s} \sum_{i \in \mathcal{I}} \lambda_s^i Y_{s,t}^i = 1 + \alpha \text{NCF}, \tag{40}$$

where NCF denotes the increase in net cash flows from innovating given by

$$\text{NCF} = \frac{1}{H}(A\delta - 1) - \left(1 - \frac{1}{H}\right)(1 - \delta). \tag{41}$$

Hence, the decision to innovate leads to constant endowment growth of each new cohort that consists of two components, that is, $\gamma - 1 = \alpha\text{NCF}$. First, only the fraction α of a birth cohort innovates in equilibrium. Second, only one out of H innovation strategies leads to success and entrepreneurial output net cost of $A\delta$ per unit of aggregate output, whereas the other strategies fail with output net cost of δ per unit of aggregate output. The terms $(A\delta - 1)/H > 0$ and $-\left(1 - \frac{1}{H}\right)(1 - \delta) < 0$ capture the spillover effect per unit of output to future cohorts from successful and unsuccessful innovations. Hence, there is positive growth for each birth cohort if this spillover effect is positive.

In the model, only newborn agents can innovate, meaning that only the fraction, ν, of endowments can be allocated towards innovations. This affects aggregate output growth in equilibrium. Specifically,

$$Y_t = Y_0 e^{\mu_Y t} \qquad \text{with} \qquad \mu_Y = \nu(\gamma - 1) = \nu\alpha\text{NCF}. \tag{42}$$

The aggregate output growth, $\mu_Y = \nu\alpha\text{NCF}$, is thus dependent on three factors: the proportion of endowments allocated towards innovation, ν; the equilibrium fraction of innovators, α; and the net cash flow (NCF) that represents the spillover per unit of output to future cohorts from both successful and unsuccessful innovations.

The left graph of Figure 2 demonstrates the impact of these factors on the growth rate. It presents μ_Y as a function of the innovation cost parameter, δ. If innovation costs surpass 60% (i.e., $\delta < 40\%$), innovation would create negative spillovers for future generations (NCF < 0), indicating a social inefficiency. The black solid line shows that the growth rate initially stagnates at zero for $\delta \leq 23\%$ values due to an absence of innovation, eliminating any negative spillovers. The red chain-dotted line then shows that the growth rate dips into negative territory when δ ranges between 23% and 40% because some agents engage in socially inefficient innovations.

However, as δ increases above 40%, innovation becomes socially beneficial (NCF > 0), generating positive spillovers. Belief diversity plays a critical role in fostering positive spillovers from innovation, thereby bolstering economic growth. In the absence of belief diversity ($\bar{\Delta} = 1$), positive spillovers would only occur if the cost is less than 0.73% (i.e., $\delta \geq A^{-\frac{1}{H}} = 99.27\%$).

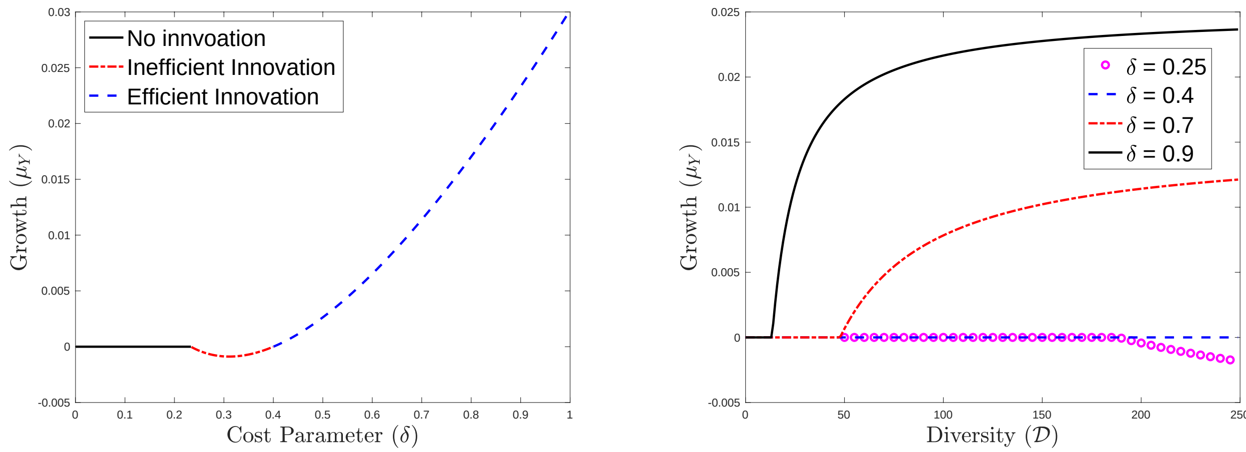


Figure 2: **Endogenous growth.** The left plot shows the endogenous output growth rate as a function of the cost parameter δ. The black solid line segment denotes the case when the fraction of innovators is zero and thus there is no growth. The red chain-dotted segment shows a range of relatively high costs of innovation for which there is no social benefit from innovating but there is still a fraction of very optimistic entrepreneurs who innovate and thus cause negative growth. The blue dashed segment shows a wide range of cost parameters for which innovations are socially beneficial and there is a lot of innovation that results in positive output growth. The right graph shows economic growth as a function of belief diversity for different cost parameters. It shows that belief diversity has an immediate positive effect on growth when it is socially valuable to innovate (solid back and chain-dotted red line) and is only socially harmful for high cost and large diversity as the circled purple line shows. In the figure we set $\nu = 0.02$, $H = 1000$, $A = 1501$, $\bar{\Delta} = 199$.

Consequently, socially efficient innovation would only occur at very low costs. Socially optimal innovation results in positive spillovers to future cohorts for all $\delta > 40\%$. However, in the absence of belief diversity, the skin-in-the-game constraint prevents many socially optimal innovations. Conversely, belief diversity alleviates this constraint and promotes positive spillovers from innovation to future generations. The right graph of Figure 2 underscores this notion by displaying the growth rate as a function of belief diversity under varying cost parameters. The black and red chain-dotted lines illustrate that growth surges with diversity when it proves socially beneficial. For the unique case of $\delta = 40\%$, growth remains unaffected by belief diversity, given that there are no spillovers to future generations ($\text{NCF} = 0$). However, for extremely high costs and large belief diversity, growth turns negative, as demonstrated by the purple circled line.

A key point to note is the importance of entrepreneurs randomizing over multiple innovation strategies, emphasizing the crucial role of belief diversity. In the absence of diverse beliefs, there is a risk of selecting non-productive innovations, leading to negative spillovers. This underlines the significance of drawing from a diverse range of innovation strategies in a winner-takes-all economy, highlighting the vital role of belief diversity in fostering economic growth.

The next proposition summarizes the derivations of aggregate output growth and shows that for socially efficient innovations economic growth is strictly increasing in diversity. The opposite is true if innovations are socially inefficient.

Proposition 4 (Belief Diversity and Economic Growth)**.** *The aggregate endowment stream of birth cohort s is $Y_{s,t} = \gamma Y_s$ and the decision to innovate leads to constant endowment growth $\gamma - 1 = \alpha NCF$. The equilibrium fraction of innovators is given in Equation (36) and both γ and NCF are given in Equation (40) and (41), respectively. Aggregate output is $Y_t = Y_0 e^{\mu_Y}$ with aggregate output growth rate μ_Y given in Equation (42). Aggregate output growth, μ_Y, increases with belief diversity, $\mathcal{D}$, for socially efficient innovations and decreases with $\mathcal{D}$ for socially inefficient ones. Specifically,*

$$\frac{\partial \mu_Y}{\partial \mathcal{D}} = \begin{cases} \frac{1}{\mathcal{D}} (1-\alpha) \, NCF \geq 0 & \text{if} \quad NCF > 0 \\ \frac{1}{\mathcal{D}} (1-\alpha) \, NCF \leq 0 & \text{if} \quad NCF < 0. \end{cases} \tag{43}$$

There is no growth if $NCF = 0$ in which case μ_Y does not depend on belief diversity.

4 Asset Pricing and Inequality

In this section, we discuss asset prices and wealth/consumption inequality.

4.1 Asset Pricing

The derivation of the risk-free rate and price-dividend ratio is similar to derivation in the exogenous growth economy discussed in Section 2. Specifically, the choice to become an entrepreneur is made just prior to entering the economy, and thus, everyone trading in the economy is facing complete markets. Agents within a birth cohort differ with respect to their wealth when entering the economy because they are either successful, unsuccessful, or not entrepreneurs. Therefore the FOCs from the exogenous growth model still hold, that is, $C^i_{s,t} = C^i_{s,s} e^{-\rho(t-s)} \frac{M_s}{M_t}$. Plugging the FOC into the aggregate resource constraint leads to

$$Y_t = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \frac{M_s}{M_t} \left(\sum_{i \in \mathcal{I}} \lambda^i_s C^i_{s,s} \right) ds, \tag{44}$$

where the measure λ^i_s of agents is given in Equation (38). The ratio of total consumption of newborns to output is

$$\beta_s = \frac{\sum_i \lambda^i_s C^i_{s,s}}{Y_s} = \frac{\gamma Y_s}{Y_s} = \gamma \qquad \forall s. \tag{45}$$

It follows from Equations (44) and (45) that

$$M_t Y_t = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \beta M_s Y_s \, ds. \tag{46}$$

The dynamics of the discount factor are $dM/M = -r\,dt$, and differentiating Equation (46) w.r.t. time t leads to the following expression for the interest rate

$$r = \rho + \mu_Y + \nu\,(1 - \beta) \tag{47}$$

We define the market portfolio with price P_t and dividend $D_t = Y_t$ as a claim on total wealth of all agents currently alive. The asset pricing results are given in the next proposition.

Proposition 5. *The equilibrium discount factor is $M_t = e^{-rt}$ with the risk-free interest rate $r = \rho$. The price-dividend ratio is $\phi = \frac{1}{\rho+\nu}$, the price of the life annuity is $\psi = \frac{1}{\rho+\nu}$, and $\beta = 1 + \alpha NCF$.*

Newborn agents innovate and impact aggregate output growth, but they do not determine the interest rate when entering the economy. The interest rate is determined by all agents currently alive, and they experience no growth in their endowments. Hence, the interest rate is the same as in an economy without growth. Consumption of newborns relative to output is $\beta = 1 + \alpha NCF = \frac{1}{\nu}\,(1 + \mu_Y)$ and the displacement effect in the interest rate due to the overlapping generations is $\nu(1-\beta) = -\mu_Y$ implying that $r = \rho + \mu_Y - \mu_Y = \rho$. The market portfolio is a claim on the total wealth of all agents currently alive, which does not include wealth that is generated by future generations. Hence, the price-dividend ratio does not depend on growth even though there is growth in the economy, and the discount rate does not depend on growth.

4.2 Wealth and consumption inequality

In this section, we examine the cross-sectional distribution of consumption and wealth in equilibrium. We know that the consumption of an agent of type i born at time s is

$$C^i_{s,t} = C^i_{s,s} = (\nu + \rho) W^i_{s,s} = \omega_i Y_s, \tag{48}$$

and hence consumption per unit of output of agent i born at time s is

$$\beta^i_{s,t} = \frac{C^i_{s,t}}{Y_t} = \omega_i e^{-\mu_Y (t-s)}. \tag{49}$$

There is no difference between consumption and wealth inequality because everybody has log-utility. We define consumption/wealth inequality as the variance across types i and birth cohorts s and derive it for positive growth in the next proposition.[4]

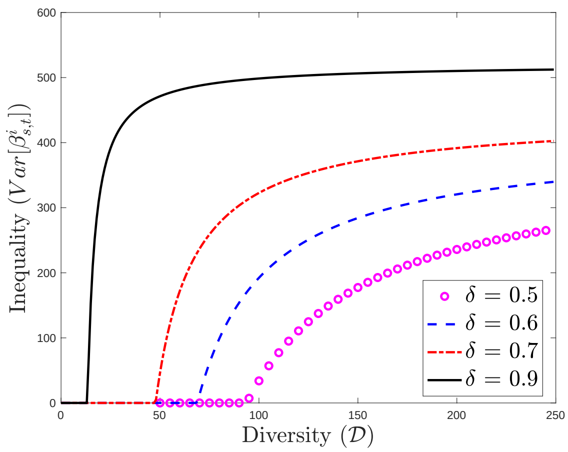
Proposition 6 (Wealth and Consumption Inequality). *Let $NCF > 0$, then*

$$\mathcal{V} \equiv Var\left[\beta^i_{s,t}\right] = \frac{\alpha}{1 + 2\alpha NCF} \left(\left(\frac{A^2}{H} + \left(1 - \frac{1}{H}\right) \right) \delta^2 - 2NCF \right). \tag{50}$$

Moreover, the consumption inequality as measured by $\mathcal{V}$ is weakly increasing in disagreement and economic growth, that is $\frac{\partial \mathcal{V}}{\partial \mathcal{D}} \geq 0$ and $\frac{\partial \mathcal{V}}{\partial \mu_Y} \geq 0$.

Belief diversity has a multifaceted impact on society, yielding both positive and negative outcomes. On one hand, it raises consumption inequality, which is typically viewed as an

[4] The cross-sectional variance does not always exist for negative growth.



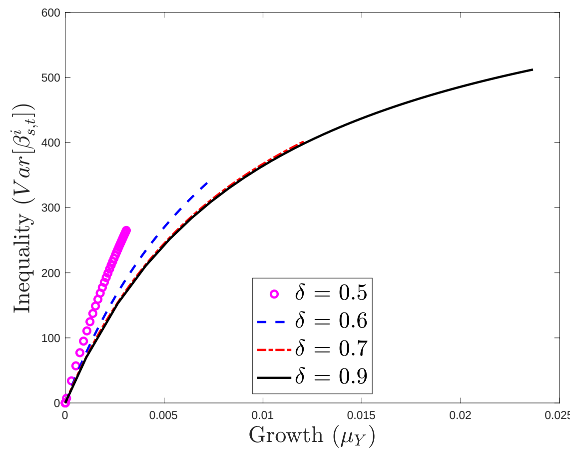


Figure 3: **Inequality.** The left plot shows inequality, $Var\left[\beta^i_{s,t}\right]$, as a function of diversity, $\mathcal{D}$, and the right graph shows it as a function of economic growth, μ_Y. All for cost parameters δ lead to social valuable innovation. Belief diversity increases inequality and it also increases growth. Hence, there is a positive relation between economic growth and belief diversity. In the figure we set $\nu = 0.02$, $H = 1000$, $A = 1501$, $\bar{\Delta} = 199$.

undesirable consequence. However, on the other hand, belief diversity plays a crucial role in fostering economic growth, which is vital for overall societal progress and prosperity. The left plot of Figure 3 demonstrates how inequality, represented by $Var\left[\beta^i_{s,t}\right]$, changes in response to diversity, $\mathcal{D}$. The right plot of Figure 3 demonstrates the relationship between inequality and economic growth, μ_Y. All cost parameters δ lead to valuable social innovation.

Interestingly, economic growth also brings benefits to future generations of non-innovators. Specifically, for non-innovators (NI), the inter-generational consumption growth over Δ years at time t is

$$\mu^{NI}_{Y,\Delta,t} \equiv \frac{1}{\Delta} \log \left(\frac{C^{NI}_{s+\Delta,t}}{C^{NI}_{s,t}} \right) = \mu_Y, \qquad \forall\, \Delta > 0 \quad \text{and} \quad t \geq s. \tag{51}$$

This implies a positive spillover of innovations to non-innovators, albeit occurring across different generations. Consequently, as long as $NCF > 0$, younger cohorts of non-entrepreneurs experience better living standards than older cohorts, while the reverse holds true if $NCF < 0$.

Innovators make optimal decisions given their beliefs, and thus they are better off in expected utility terms when they choose to innovate. However, not all innovators can be correct since only one out of the H possible ways of innovating is successful. Hence, one might argue that a social planner should consider the ex-post utility based on a success probability of $\frac{1}{H}$, that is, the consensus probability that a specific path leads to success. The next proposition derives the ex-post utility gain/loss from innovating based on a representative cohort.

Proposition 7. *Let $EU_s^{e,0}$ denote the lifetime expected utility of an entrepreneur based on the success probability of $1/H$ and U_s^{se} denote the lifetime utility from not innovating. The difference between the utility from innovating and not innovating is*

$$dU \equiv EU_s^{e,0} - U_s^{se} = \frac{1}{\rho+\nu}\left(\frac{1}{H}\log(A) + \log(\delta)\right). \tag{52}$$

Moreover, $dU < 0$ for $\delta < \delta^{**} = A^{-\frac{1}{H}}$.

Proposition 7 shows that from a social planner's viewpoint, the expected utility of innovators is less than the utility of non-innovators, unless the costs of innovation are sufficiently low, specifically when $\delta > \delta^{**} = A^{-\frac{1}{H}}$. In the context of our baseline model, this threshold cost is less than 0.73%, or equivalently, $\delta > 99.27\%$. Interestingly, while a rise in belief diversity fuels the likelihood of innovation, spurring potential economic growth, it concurrently diminishes the welfare of innovators from a social planner's perspective.

5 Entrepreneurial Risk Sharing

In this section, we study two different mechanisms to share entrepreneurial risk: taxes and a venture capital fund.

5.1 Government Intervention through Taxation

In this section, we study three distinct tax scenarios. First, we examine a lump-sum tax redistributed within birth cohorts. This tax serves as a risk-sharing mechanism, fostering innovation by reducing the risk burden on individual entrepreneurs. However, this model leads to a decrease in economic growth due to a reduction in entrepreneurial capital. Given the equal tax distribution within the same birth cohorts, this approach leaves asset prices unaffected. Next, we investigate a flat tax with redistribution across birth cohorts. Here, taxes are utilized to finance innovation and are evenly distributed across different generations. This strategy results in a wealth transfer from younger generations to older ones, particularly in a growing economy. As a consequence, interest rates and valuation ratios are impacted. Lastly, we consider a flat tax with redistribution within birth cohorts combined with a costly effort to innovate. In this scenario, a trade-off materializes, balancing risk-sharing against the necessity for entrepreneurs to invest in costly innovative efforts. Consequently, a hump-shaped economic growth trend emerges when plotted against tax rates, highlighting an optimal tax rate that stimulates maximum economic growth.

5.1.1 Lump-Sum Tax with Redistribution within Birth Cohort

Consider birth cohort s with endowment Y_s that pays the lump-sum tax $\mathcal{T}_s = \tau Y_s$ over their lifetime. We refer to the tax amount as a lump sum because it does not depend on the endowment post-innovation. The remaining endowment stream $(1-\tau)Y_s$ can be used to innovate, and the taxes levied by the government are redistributed within the birth cohort s. Hence, the endowment stream and its value after the innovation decision is made for all birth cohorts $s \leq t \leq \tau$ and agent type's $i \in \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$ are

$$Y^i_{s,t} = \omega_i(1-\tau)Y_s + \tau Y_s, \tag{53}$$

$$W^i_{s,t} = \left(\omega_i(1-\tau) + \tau\right)\psi Y_s, \tag{54}$$

where $\psi = 1/(\nu + r)$ is the constant price of a life annuity and ω_i captures the different wealth of successful, unsuccessful, and non-innovators given in Equation (21). Taxes collected within a birth cohort are redistributed within a birth cohort, and thus, the budget constraint of the government is satisfied. Effectively, the tax imposed forces agents to take less entrepreneurial risk.

An agent born at time s will choose to become an entrepreneur if the expected lifetime utility from innovating is higher than the lifetime expected utility from not innovating. Hence, agents become entrepreneurs if their success probability exceeds the threshold

$$\frac{\Delta^*_\tau}{H} = \frac{\log\left(W^{\text{ne}}_{s,s}/W^{\text{ue}}_{s,s}\right)}{\log\left(W^{\text{se}}_{s,s}/W^{\text{ue}}_{s,s}\right)} = \frac{-\log\left(\tau + \delta(1-\tau)\right)}{\log\left((\tau + A\delta(1-\tau))/(\tau + \delta(1-\tau))\right)}. \tag{55}$$

The likelihood of innovation is

$$\alpha_\tau = \mathbb{P}\left(\Delta > \Delta^*_\tau\right) = \frac{\bar{\Delta} - \Delta^*_\tau}{\bar{\Delta} - 1}. \tag{56}$$

The left plot of Figure 4 shows the equilibrium fraction of innovators α_τ as a function of the tax rate τ for different cost parameters δ. The equilibrium fraction of innovators is strictly increasing in taxes for all cost parameters that would lead to innovation if no taxes are collected, that is, $\delta > 23.33\% = A^{-\bar{\Delta}/H}$. For a higher cost, it is weakly increasing (e.g. $\delta = 0.1$). Strikingly, a moderate tax rate of 30% leads to an innovation likelihood of 12% in equilibrium despite a high cost of 90% (yellow line with stars). To determine economic growth, we plug the individual

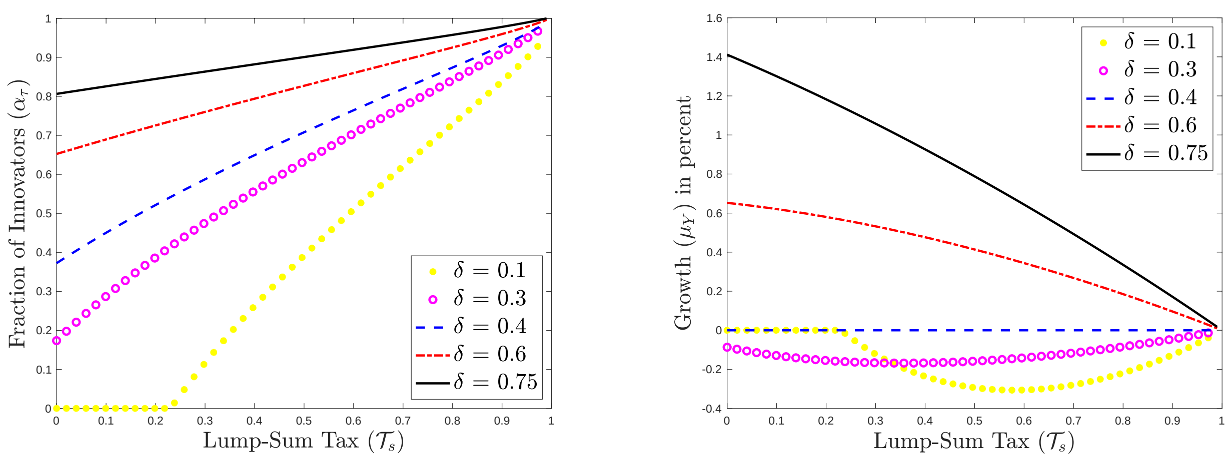


Figure 4: **Entrepreneurial Risk Sharing with Lump Sum Taxes.** The left plot shows the equilibrium fraction of innovators α_τ, and the right plot shows growth in equilibrium as a function of the lump-sum taxes ($\mathcal{T}_s$). The likelihood of innovation increases in taxes, but economic growth decreases in taxes if innovation is socially optimal and is hump-shaped otherwise. In the figure we set $\nu = 0.02$, $H = 1000$, $A = 1501$, $\bar{\Delta} = 199$, and $Y_s = 1$.

endowments given in Equation (53) into the aggregate resource constraint and get

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \sum_{i\in\mathcal{I}} \lambda_s^i Y_{s,t}^i \, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma_\tau Y_s \, ds, \tag{57}$$

where λ_t^i is the measure of type i agents as defined in Equation (39) of the previous section, and

$$\gamma_\tau = \tau + (1-\tau)\left(1 + \alpha_\tau NCF\right). \tag{58}$$

The increase in net cash flows from innovating, NCF, is given in Equation (41) and does not depend on taxes. Hence, economic growth is

$$\mu_Y^\tau = \nu(\gamma_\tau - 1) = \nu(1-\tau)\alpha_\tau NCF. \tag{59}$$

The right plot of Figure 4 shows economic growth as a function of the tax rate τ for different cost parameters δ. Growth is strictly decreasing in the tax rate τ when innovation is socially valuable without taxes, that is, if $\delta > H/(H + A - 1) = 40\%$. In this case, the increase in the likelihood of innovation due to improved entrepreneurial risk sharing does not outweigh the reduction in entrepreneurial capital because the lump sum taxes levied by the government are not deployed for innovation. For innovations that are not socially optimal, $\delta < 40\%$ growth is hump-shaped.

5.1.2 Flat tax with redistribution across cohorts

In the case without taxes, the lifetime endowment (after the choice of innovating) of an agent born at time t of type i is $\omega_i Y_s$, where ω_i captures the different wealth of successful, unsuccessful, and non innovators given in Equation (21). In this subsection, we consider the case of a flat tax rate and tax redistribution within and across cohorts.

It immediately follows from Equation (38) that the total tax revenue, $\mathcal{T}_t$, at time t is

$$\mathcal{T}_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \sum_{i \in \mathcal{I}} \lambda_s^i \tau w_i Y_s \, ds = \tau Y_t, \tag{60}$$

where λ_t^i is the measure of type i agents as defined in Equation (39). The tax revenues are equally spread among all agents, hence the total (post) transfers endowment, $Y_{s,t}^i$, for all birth cohorts $s \leq t \leq \tau$ and agent type's $i \in \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$ is

$$Y_{s,t}^i = \omega_i (1 - \tau) Y_s + \tau Y_t. \tag{61}$$

In contrast to the previous tax example taxes are equally distributed among all cohorts. Hence, there is a wealth transfer between generations that the previous examples did not consider. If there is growth, then this implies a net transfer from younger generations to older generations. This transfer impacts the risk-free rate and valuation because tax transfers depend on total output and thus individual endowments that include the tax transfer grow. This is fundamentally different from the previous sections. Hence, the total wealth, $W_{s,s}$, of an agent born at time s of type i is

$$W_{s,s}^i = \omega_i (1 - \tau) \psi(r) Y_s + \tau \phi(\mu_Y, r) Y_s \tag{62}$$

where the valuation ratios $\psi(r) = 1/(\nu + r)$ and $\phi(\mu_Y, r) = 1/(\nu + r - \mu_Y)$ are constants. The reasons for the different valuation ratios is that the tax transfers are a claim to total output whereas individual endowments before transfers do not grow. Consequently, the two claims have different durations. As before, we can solve for the innovation threshold, $\Delta_\tau^*(\mu_Y, r)$, and the fraction of newborns that chose to innovate, $\alpha_\tau(\mu_Y, r)$. Specifically,

$$\frac{\Delta_\tau^*(\mu_Y, r)}{H} = \frac{\log\left(W_{s,s}^{\text{ne}} / W_{s,s}^{\text{ue}}\right)}{\log\left(W_{s,s}^{\text{se}} / W_{s,s}^{\text{ue}}\right)} = \frac{\log\left(\frac{(1-\tau)\psi(r) + \tau\phi(\mu_Y, r)}{(1-\tau)\delta\psi(r) + \tau\phi(\mu_Y, r)}\right)}{\log\left(\frac{(1-\tau)A\delta\psi(r) + \tau\phi(\mu_Y, r)}{(1-\tau)\delta\psi(r) + \tau\phi(\mu_Y, r)}\right)} \tag{63}$$

and it follows that the likelihood of innovation is

$$\alpha_\tau(\mu_Y, r) = \mathbb{P}\left(\Delta > \Delta^*\right) = \frac{\bar{\Delta} - \Delta^*}{\bar{\Delta} - 1}. \tag{64}$$

The innovation likelihood depends on valuation ratios which depend on the risk-free rate and economic growth which depends on the innovation likelihood. Hence, we have a fixed point problem. The next proposition presents the equation that the interest rate and growth rate have to satisfy in equilibrium.

Proposition 8. *The risk-free rate r and the growth rate μ_Y satisfy the two equations*

$$\mu_Y = \nu \alpha_\tau(\mu_Y, r) NCF \tag{65}$$

$$r = \rho + \mu_Y + \nu\left(1 - \beta_\tau(\mu_Y, r)\right) \tag{66}$$

with NCF given in Equation (41) and valuation ratios $\psi(r) = 1/(\nu + r)$ and $\phi(\mu_Y, r) = 1/(\nu + r - \mu_Y)$. Moreover, the average consumption of newborns relative to output is

$$\beta_\tau = (\rho + \nu)\left(\left(1 + \alpha_\tau NCF\right)(1 - \tau)\,\psi + \tau\phi\right) \tag{67}$$

Special cases: $r = \rho$ when $\tau = 0$ and $r = \rho + \mu_Y$ when τ approaches one.

The likelihood of innovation is increasing in the tax rate because the redistribution lowers entrepreneurial risk as discussed in the previous section. This leads to growth when it is socially desirable as shown in the left plot of Figure 5 by the solid black and chain-dotted red line. In the case of socially undesirable innovation, an increase in the tax rate lowers growth as shown by the circled purple and the starred yellow lines. The right plot of Figure 5 shows that the interest rate, r, is strictly increasing in the flat tax rate τ when there is positive growth and decreasing when there is negative growth. When the flat tax rate approaches one, then we have perfect sharing of entrepreneurial risk and the same interest rate as in the exogenous growth case without any heterogeneity, that is, $r = \rho + \mu_Y$.

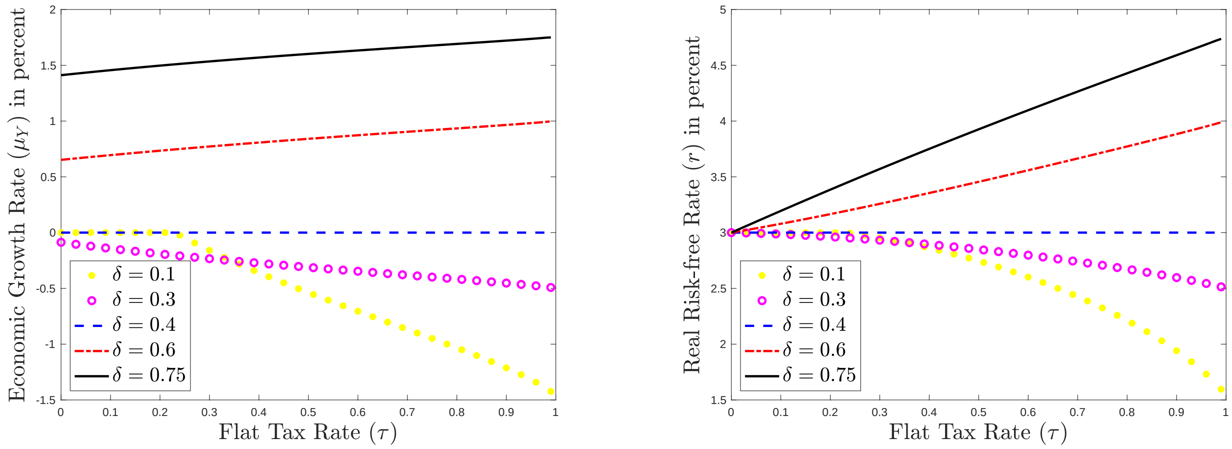


Figure 5: **Entrepreneurial Risk Sharing with Flat Taxes and Redistribution across Cohorts.** The left plot shows economic growth and the right plot shows interest rate in equilibrium as a function of the flat tax rate (τ). An increase in the tax rate raises economic growth and thus interest rates when innovation is socially desirable. The opposite is true when it is socially undesirable. In the figure we set $\nu = 0.02$, $\rho = 0.03$, $H = 1000$, $A = 1501$, and $\bar{\Delta} = 199$.

5.1.3 Flat tax redistribution within birth cohorts and costly effort to innovate

In contrast to the previous sections of the paper, we explicitly model the disutility from innovating in this section and discuss the effects of a flat tax with redistribution within birth cohorts on economic growth.[5] Specifically, consider the model from Section 3 and suppose an agent has to put in effort when innovating. If she exerts effort, then the perceived probability of success from choosing a specific path is Δ/H and her disutility from exerting effort is $\frac{1}{\nu+\rho}\log(1+e)$ with $e \geq 0$. If $e = 0$, then there is no disutility from exerting effort as in Section 3. Hence, the expected utility from innovating given in Equation (34) changes to

$$\begin{aligned} \mathrm{E}^i\left[U_s^{i,e}\right] &= \frac{1}{\rho+\nu}\mathrm{E}^i\left[\log\left(W_{s,s}^i\right)\right] - \frac{1}{\rho+\nu}\log(1+e) + \hat{U}_s \\ &= \frac{1}{\rho+\nu}\left(\frac{\Delta}{H}\log\left(\frac{W_{s,s}^{\text{se}}}{W_{s,s}^{\text{ue}}}\right) + \log\left(\frac{W_{s,s}^{\text{ue}}}{1+e}\right)\right) + \hat{U}_s, \end{aligned} \tag{68}$$

where $\hat{U}_s$ is independent of the choice to become an entrepreneur and given in Equation (133) of the Appendix. Let $\omega_i Y_s$ denote the pretax endowment stream of a type i agent of birth cohort

[5]We thank our discussant at the 20th Macro Finance Society Workshop, Stavros Panageas, for suggesting this example.

s. Agent i is taxed at the rate τ after her innovation decision is made and all the uncertainty about it is resolved and hence her after tax endowment stream before tax redistribution is $(1-\tau)\omega_i Y_s$. The total tax revenue of birth cohort s is

$$\mathcal{T}_s = \tau \sum_{i \in \mathcal{I}} \lambda_s^i w_i Y_s = \tau \gamma_{\tau,e} Y_s, \qquad \gamma_{\tau,e} = \alpha_{\tau,e}\delta\left(\frac{A}{H} + \left(1 - \frac{1}{H}\right)\right) + (1-\alpha_{\tau,e}), \tag{69}$$

where λ_t^i is the measure of agents of type i given in Equation (39). The tax revenue of birth cohort s are equally spread among all agents within birth cohort s. Hence, the time-t total (post) transfer endowment of birth cohort s is

$$Y_{s,t}^i = \omega_i(1-\tau)Y_s + \tau\gamma_{\tau,e}Y_s. \tag{70}$$

The tax amount that is equally distributed within a birth cohort depends on the fraction of innovators $\alpha_{\tau,e}$ which is decreasing in disutility of effort e and thus there is a trade-off between the improved risk sharing and the reduced incentives from taxation. However, there is no tax redistribution across birth cohorts and thus the risk-free rate and valuation ratios are unaffected by the tax rate τ. Hence, the total wealth, $W_{s,s}$, of an agent born at time s of type i is

$$W_{s,s}^i = \left(\omega_i(1-\tau) + \tau\gamma_{\tau,e}\right)\psi Y_s, \tag{71}$$

where the valuation ratio $\psi = 1/(\nu + r)$ is constant. As before, we can solve for the innovation threshold, $\Delta_{\tau,e}^*$, and the fraction of newborns that chose to innovate, $\alpha_{\tau,e}$. Specifically,

$$\frac{\Delta_{\tau,e}^*}{H} = \frac{\log\left((1+e)W_{s,s}^{\text{ne}}/W_{s,s}^{\text{ue}}\right)}{\log\left(W_{s,s}^{\text{se}}/W_{s,s}^{\text{ue}}\right)} = \frac{\log\left(\frac{(1+e)((1-\tau)+\tau\gamma_{\tau,e})}{(1-\tau)\delta+\tau\gamma_{\tau,e}}\right)}{\log\left(\frac{(1-\tau)A\delta+\tau\gamma_{\tau,e}}{(1-\tau)\delta+\tau\gamma_{\tau,e}}\right)} \tag{72}$$

and it follows that the fraction of innovators and economic growth in equilibrium are

$$\alpha_{\tau,e} = \frac{\bar{\Delta} - \Delta^*_{\tau,e}}{\bar{\Delta} - 1} \tag{73}$$

$$\mu_Y^{\tau,e} = \nu(\gamma_{\tau,e} - 1) = \nu\alpha_{\tau,e} NCF, \tag{74}$$

where NCF is independent of effort and taxes, and given in Equation (41). There is no redistribution across cohorts and thus taxes do not affect the risk-free rate and the valuation ratios, that is, they are the same as in Proposition 5. Hence, they are also not affected by effort.

There is a trade-off within a birth cohort between improved entrepreneurial risk sharing with an increase in the flat tax and the resulting decrease in incentives to become an entrepreneur. This trade-off is illustrated in Figure 6 that shows the equilibrium growth rate for different effort levels as a function of tax rate in the left plot and as a function of disagreement in the right plot. The cost parameter in both graphs is $\delta = 0.6$ and we set the tax rate to $\tau = 0.5$ in the right plot. The solid black line illustrates that the economic growth rises with the tax rate when effort involves no disutility due to better risk sharing. However, when there is disutility from exerting effort, high tax rates suppress innovation incentives, overruling risk-sharing benefits, and creating a non-monotonic link between the fraction of innovators and the tax rate (as shown by the red chain-dotted, blue dashed, purple circled, and yellow star lines). The right plot shows that diversity in beliefs raises growth for all effort levels as soon as diversity is large enough such that there is innovation in equilibrium. Otherwise there is no growth.

5.2 Venture Capital

Consider birth cohort s with endowment Y_s that can invest the fraction θ of its endowment into a fund that invests in all H projects. There is a fund for each birth cohort that allows entrepreneurs of the birth cohort to completely diversify all the idiosyncratic entrepreneurial

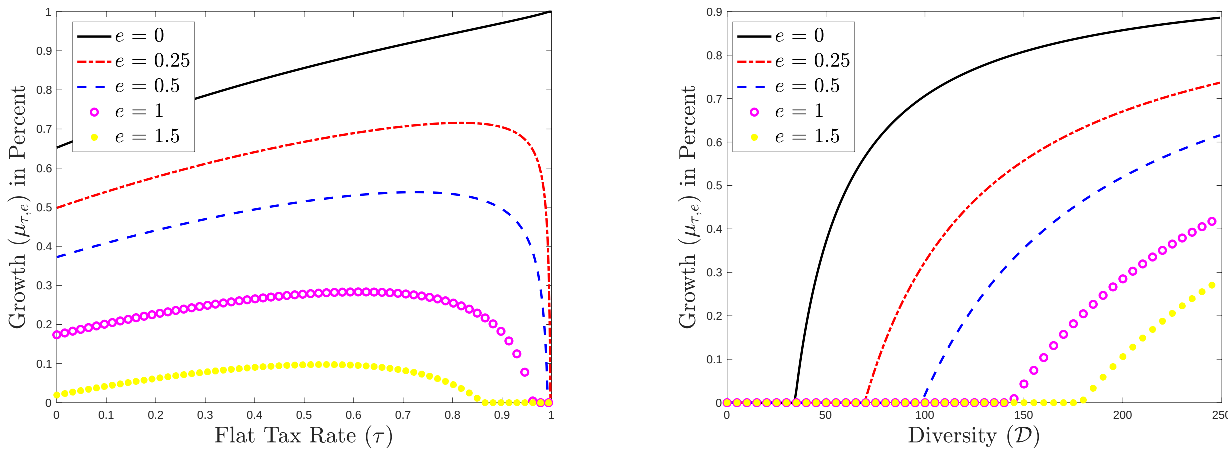


Figure 6: **Tradeoff between Entrepreneurial Risk Sharing and Innovation Incentives trough a Flat Tax with Redistribution within Cohorts.** This Figure shows economic growth in equilibrium μ_Y as a function of the flat tax rate τ in the left plot and as a function of belief diversity in the right plot. We set the cost parameter to $\delta = 0.6$ leading to positive NCF in both graphs and the tax rate is $\tau = 0.5$ in the right graph. The other parameters are set to $\nu = 0.02$, $\rho = 0.03$, $H = 1000$, $A = 1501$, and $\bar{\Delta} = 199$.

risk. Hence, the fund pays the certain amount

$$\left(A\frac{1}{H} + \left(1 - \frac{1}{H}\right)\right)\delta Y_s = (1 + NCF)Y_s, \tag{75}$$

where NCF is given in Equation (41). Suppose $\delta > 40\%$ and thus the return of the fund, NCF, is always positive. Every agent invests θY_s in this fund and decides whether to use the remaining fraction $1 - \theta$ to become an entrepreneur.[6] Hence, the endowment stream and its value after the innovation decision is made for all $s \leq t \leq \tau$ are

$$Y^i_{s,t} = \omega_i(1-\theta)Y_s + \theta(1 + NCF)Y_s \qquad \text{and} \qquad W^i_{s,t} = Y^i_{s,t}\psi, \tag{76}$$

where $\psi = 1/(\nu + r)$ is the constant price of a life annuity and ω_i captures the different wealth of successful, unsuccessful, and non-innovators given in Equation (21). An agent born at time s will choose to become an entrepreneur if the expected lifetime utility from innovating is higher than the lifetime expected utility from not innovating. Hence, agents become entrepreneurs if

[6]The fraction θ is exogenously given but can be endogenized by introducing costly effort and moral hazard.

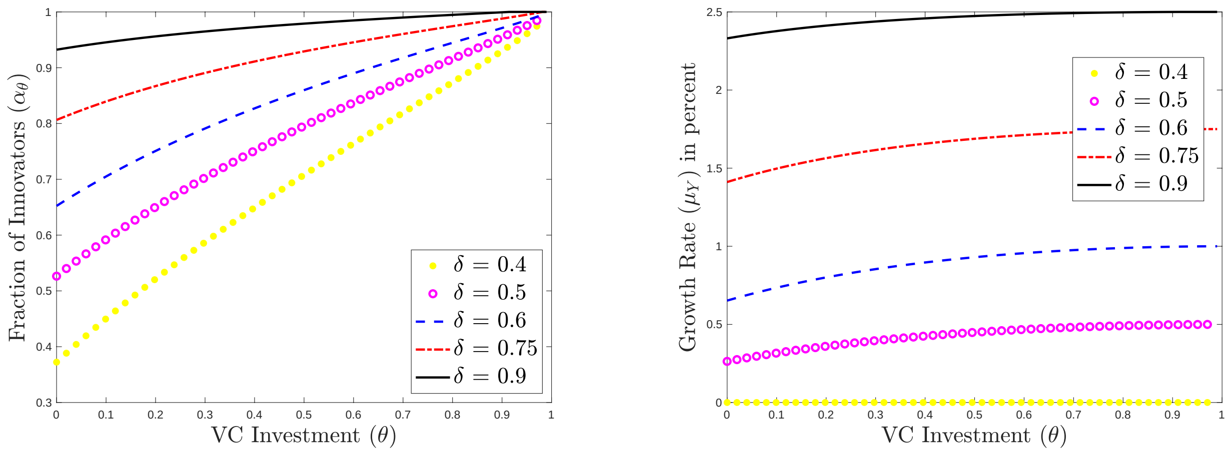


Figure 7: **Entrepreneurial Risk Sharing with a Venture Capital Fund.** The left plot shows the equilibrium fraction of innovators α_θ and the right plot shows growth in equilibrium as a function of the VC investment (θ). If $\delta > 40\%$ and thus NCF is strictly positive, then an increase in VC investment lowers the entrepreneurial risk and thus increases the likelihood of innovation and economic growth. In the figure we set $\nu = 0.02$, $H = 1000$, $A = 1501$ and $\bar{\Delta} = 199$.

their probability of success exceeds the threshold

$$\frac{\Delta_\theta^*}{H} = \frac{\log\left(W_{s,s}^0 / W_{s,s}^L\right)}{\log\left(W_{s,s}^H / W_{s,s}^L\right)} = \frac{\log\left((1+\theta NCF) \,/\, (\theta(1+NCF)+(1-\theta)\delta)\right)}{\log\left((\theta(1+NCF)+(1-\theta)A\delta) \,/\, (\theta(1+NCF)+(1-\theta)\delta)\right)}. \tag{77}$$

The likelihood of innovation in equilibrium is

$$\alpha_\theta = \mathbb{P}\left(\Delta > \Delta_\theta^*\right) = \frac{\bar{\Delta} - \Delta_\theta^*}{\bar{\Delta} - 1}. \tag{78}$$

The left plot of Figure 7 shows the equilibrium fraction of innovators α_θ as a function of the VC investment θ for different cost parameters δ. The equilibrium fraction of innovators is strictly increasing in the VC investment for all cost parameters $\delta > 40\%$ so that NCF is strictly positive because the VC investment lowers entrepreneurial risk. To determine economic growth we plug Equation (76) into the aggregate resource constraint. Specifically,

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \sum_{i\in\mathcal{I}} \lambda_s^i Y_{s,t}^i \, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma_\theta Y_s \, ds, \tag{79}$$

where λ_t^i is the measure of agents of type i given in Equation (39) and

$$\gamma_\theta = 1 + (1 - (1-\theta)(1-\alpha_\theta))\, NCF. \tag{80}$$

Hence, economic growth is

$$\mu_Y^\theta = \nu(\gamma_\theta - 1) = \nu\,(1 - (1-\theta)(1-\alpha_\theta))\, NCF. \tag{81}$$

The right plot of Figure 7 shows economic growth as a function of the VC investment θ for different cost parameters δ. If $\delta > 40\%$ and thus NCF is strictly positive, then an increase in VC investment lowers the entrepreneurial risk and thus increases economic growth.

6 Conclusion

We study an equilibrium model with diverse perspectives on the likelihood of innovation success. We show that such diversity fuels aggregate economic growth and overcomes market failures that could otherwise occur in an equilibrium without belief diversity. This growth enhancement through diversity, however, comes at the expense of an increase in wealth and consumption inequality as only a few entrepreneurs succeed while the majority do not. Through tax schemes and venture capital funds, risk sharing among entrepreneurs is improved, thereby increasing the likelihood for innovation. This entrepreneurial risk reduction bolsters growth unless high taxation drains entrepreneurial capital. Redistributing taxes across cohorts mitigates inequality and potentially elevates interest rates. However, introducing costly effort disincentivizes innovation when the tax burden on entrepreneurs increases. Therefore, a trade-off emerges between risk-sharing and exerting costly effort, resulting in a hump-shaped economic growth when graphed against tax rates.

Understanding the role of diversity in beliefs in fostering economic growth is crucial for several reasons. First, it promotes innovation and entrepreneurship. Belief diversity is often the driving force behind innovation. Entrepreneurs with differing views, assumptions, and outlooks engage in risky ventures, experimenting with different ideas, which often leads to breakthroughs and progress. This entrepreneurial activity is critical to the dynamism of an economy and its long-term growth. Second. it broadens economic possibilities. When there is belief diversity, society is effectively "drawing from the entire distribution of ideas." Hence, multiple avenues for innovation are explored, thus broadening the scope of economic possibilities and enhancing the prospects for economic growth. Third, it informs economic policy. Understanding belief diversity can inform policy aimed at fostering innovation and economic growth. For instance, policies that encourage diversity of thought and risk-taking could be implemented to stimulate entrepreneurial activity and thus economic growth. Fourth, it leads to wealth and consumption inequalities. Our findings about the relationship between diversity in beliefs, economic growth, and the rise of wealth and consumption inequalities could stimulate further research on these topics. This is important given the ongoing debates about economic inequality.

We also provide new insights into the "entrepreneurial puzzle". By proposing a new angle—diversity in beliefs—we enrich our understanding of what motivates entrepreneurs to take on substantial risk and innovate. Further, by investigating the potential costs and benefits of diversity in beliefs (like wealth inequality and economic growth), our research provides a more nuanced understanding of the dynamics of economic development. Finally, the unique model introduced in this study, which employs diverse beliefs among agents and a "skin in the game" constraint, contributes a new methodological approach that future research in the field can build upon. This can open up new avenues for further research and contribute to the body of knowledge on entrepreneurship and economic growth.

References

Acharya, V. V., L. H. Pedersen, 2005. Asset pricing with liquidity risk. Journal of financial Economics 77(2), 375–410.

Aghion, P., P. Howitt, 1992. A model of growth through creative destruction. Econometrica 60(2), 323–351.

Akcigit, U., S. Baslandze, S. Stantcheva, 2016. Taxation and the international mobility of inventors. American Economic Review 106(10), 2930–2981.

Akcigit, U., S. Stantcheva, 2020. Taxation and Innovation: What Do We Know?. Unpublished working paper. National Bureau of Economic Research.

Andrei, D., B. I. Carlin, 2023. Schumpeterian Competition in a Lucas Economy. Journal of Economic Theory p. 105613.

Basak, S., 2000. A Model of Dynamic Equilibrium Asset Pricing with Heterogeneous Beliefs and Extraneous Risk. Journal of Economic Dynamics and Control 24, 63–95.

Basak, S., 2005. Asset Pricing with Heterogeneous Beliefs. Journal of Banking and Finance 29, 2849–2881.

Berrada, T., 2006. Incomplete Information, Heterogeneity, and Asset Pricing. Journal of Financial Econometrics 4, 136–160.

Bhamra, H. S., R. Uppal, 2014. Asset Prices with Heterogeneity in Preferences and Beliefs. Review of Financial Studies 27, 519–580.

Blanchard, O., 1985. Debt, Deficits, and Finite Horizons. Journal of Political Economy 93, 223–247.

Borovička, S., 2019. Survival and Long-Run Dynamics with Heterogeneous Beliefs under Recursive Preferences. Journal of Political Economy forthcoming pp. 1–48.

Buraschi, A., A. Jiltsov, 2006. Model Uncertainty and Option Markets with Heterogeneous Beliefs. Journal of Finance 61, 2841–2897.

Buraschi, A., F. Trojani, A. Vedolin, 2014. When Uncertainty Blows in the Orchard: Comovement and Equilibrium Volatility Risk Premia. Journal of Finance 69, 101–137.

Chien, Y., H. Cole, H. Lustig, 2016. Implications of heterogeneity in preferences, beliefs and asset trading technologies in an endowment economy. Review of Economic Dynamics 20, 215–239.

Collin-Dufresne, P., M. Johannes, L. A. Lochstoer, 2017. Asset Pricing when 'This Time is Different'. Review of Financial Studies 30, 505–535.

Constantinides, G. M., J. B. Donaldson, R. Mehra, 2002. Junior can't borrow: a new perspective on the equity premium puzzle. The Quarterly Journal of Economics 117(1), 269–296.

Cox, J., C. Huang, 1989. Optimal Consumption and Portfolio Policies when Asset Prices Follow a Diffusion Process. Journal of Economic Theory 83, 33–83.

Croce, M. M., H. Kung, T. T. Nguyen, L. Schmid, 2012. Fiscal policies and asset prices. Review of Financial Studies 25(9), 2635–2672.

Croce, M. M., T. T. Nguyen, L. Schmid, 2012. The market price of fiscal uncertainty. Journal of Monetary Economics 59(5), 401–416.

Cujean, J., M. Hasler, 2017. Why Does Return Predictability Concentrate in Bad Times?. Journal of Finance 72, 2717?2758.

Cvitanic, J., E. Jouini, S. Malamud, C. Napp, 2012. Financial Markets Equilibrium with Heterogeneous Agents. Review of Finance 16, 285–321.

Cvitanic, J., S. Malamud, 2011. Price Impact and Portfolio Impact. Journal of Financial Economics 100, 201–225.

David, A., 2008. Heterogeneous Beliefs, Speculation, and the Equity Premium. Journal of Finance 63(1), 41–83.

Detemple, J. B., S. Murthy, 1994. Intertemporal Asset Pricing with Heterogeneous Beliefs. Journal of Economic Theory 62, 294–320.

Dumas, B., A. Kurshev, R. Uppal, 2009. Equilibrium Portfolio Strategies in the Presence of Sentiment Risk and Excess Volatility. Journal of Finance 64, 579–629.

Ehling, P., M. Gallmeyer, C. Heyerdahl-Larsen, P. Illeditsch, 2018. Disagreement about Inflation and the Yield Curve. Journal of Financial Economics 127 (3), 459 – 484.

Ehling, P., A. Graniero, C. Heyerdahl-Larsen, 2018. Asset Prices and Portfolio Choice with Learning from Experience. Review of Economic Studies 85 (3), 1752 – 1780.

Friedel, R., P. Israel, 2010. Edison's Electric Light: The Art of Invention (Revised ed.). The Johns Hopkins University Press, .

Gârleanu, N., L. Kogan, S. Panageas, 2012. Displacement risk and asset returns. Journal of Financial Economics 105, 491–510.

Gârleanu, N., S. Panageas, 2015. Young, Old, Conservative and Bold: The Implications of Heterogeneity and Finite Lives for Asset Pricing. Journal of Political Economy 123, 670–685.

Gârleanu, N., S. Panageas, 2021. What to expect when everyone is expecting: Self-fulfilling expectations and asset-pricing puzzles. Journal of Financial Economics 140(1), 54–73.

Garleanu, N., S. Panageas, D. Papanikolaou, J. Yu, 2016. Drifting apart: The pricing of assets when the benefits of growth are not shared equally. Working paper, UC Berkeley.

Garleanu, N., S. Panageas, J. Yu, 2012. Technological growth and asset pricing. The Journal of Finance 67(4), 1265–1292.

Gomes, F., A. Michaelides, 2005. Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence. Journal of Finance 60(2), 869–869.

Gomez, M., 2016. Asset prices and wealth inequality. Unpublished paper: Princeton.

Grossman, G. M., E. Helpman, 1993. Innovation and growth in the global economy. MIT press, .

Haddad, V., P. Ho, E. Loualiche, 2018. Efficient Bubbles?. in *Paris December 2018 Finance Meeting EUROFIDAI-AFFI*.

Harrison, J., D. Kreps, 1978. Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations. Quarterly Journal of Economics 92, 323–336.

Heyerdahl-Larsen, C., P. Illeditsch, 2021. The Market View. Working Paper.

Heyerdahl-Larsen, C., P. Illeditsch, 2023. Demand Disagreement. Working Paper.

Jouini, E., C. Napp, 2007. Consensus Consumer and Intertemporal Asset Pricing with Heterogeneous Beliefs. Review of Economic Studies 74, 1149 – 1174.

Karatzas, I., S. E. Shreve, 1998. Methods of Mathematical Finance. Springer, .

Kogan, L., D. Papanikolaou, N. Stoffman, 2013. Winners and losers: Creative destruction and the stock market. Unpublished working paper. National Bureau of Economic Research.

Kogan, L., D. Papanikolaou, N. Stoffman, 2019. Left behind: Creative destruction, inequality, and the stock market. Journal of Political Economy, forthcoming pp. 1–48.

Kung, H., 2015. Macroeconomic linkages between monetary policy and the term structure of interest rates. Journal of Financial Economics 115(1), 42–57.

Kung, H., L. Schmid, 2015. Innovation, growth, and asset prices. The Journal of Finance 70(3), 1001–1037.

Lin, X., D. Palazzo, F. Yang, 2017. The risks of old age: Asset pricing implication of technology adoption. Working Paper, Ohio State University.

Miller, E. M., 1977. Risk, Uncertainty, and Divergence of Opinion. Journal of Finance XXXII(4), 1151–1168.

Moskowitz, T. J., A. Vissing-Jørgensen, 2002. The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?. American Economic Review pp. 745–778.

Opp, C. C., 2019. Venture capital and the macroeconomy. The Review of Financial Studies 32(11), 4387–4446.

Pástor, L., P. Veronesi, 2016. Income inequality and asset prices under redistributive taxation. Journal of Monetary Economics 81, 1–20.

Pastor, L., P. Veronesi, 2018. Inequality aversion, populism, and the backlash against globalization. Unpublished working paper. National Bureau of Economic Research.

Scheinkman, J. A., W. Xiong, 2003. Overconfidence and Speculative Bubbles. Journal of Political Economy 111(6), 1183–1219.

Simsek, A., 2013a. Belief disagreements and collateral constraints. Econometrica 81(1), 1–53.

Simsek, A., 2013b. Speculation and risk sharing with new financial assets. The Quarterly journal of economics 128(3), 1365–1396.

Xiong, W., H. Yan, 2010. Heterogeneous Expectations and Bond Markets. Review of Financial Studies 23, 1433–1466.

Zapatero, F., 1998. Effects of Financial Innovations on Market Volatility when Beliefs are Heterogeneous. Journal of Economic Dynamics and Control 22, 597–626.

A Proofs

In this Appendix, we provide comprehensive proofs for the propositions presented in the main text. To ensure clarity and completeness, these proofs incorporate additional details of the theoretical arguments which were necessarily omitted in the main text for the sake of brevity and focus. Each subsection here aligns with a section of the main text.

A.1 Exogenous growth

Proof 1 (Proof of Proposition 1)**.** *We start by determining aggregate output growth, that is, we demonstrate the validity of Equation (4). Specifically, we substitute the individual endowment of birth cohort s, that is, $Y_{s,t} = \gamma Y_s$ into the aggregate resource constraint given in Equation (2) which leads to*

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t}\, ds = \gamma \int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_s\, ds. \tag{82}$$

In Equation (82) we integrate over the birth cohort variable s and subsequently use Leibniz rule to determine the derivative of this integral w.r.t. time t, that is, we treat time t as a parameter. Hence, the dynamics of aggregate output Y_t are

$$dY_t = \left(-\nu \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma Y_s \, ds + \nu \gamma Y_t \right) dt = \nu(\gamma - 1) Y_t \, dt. \tag{83}$$

This provides the required proof for Equation (4), that is, aggregate output growth equals $\mu_Y = \nu(\gamma - 1)$.

After an agent's birth, the agent faces a dynamically complete security market consisting of the real risk-free asset B_t and the life insurance/annuity contract $\mathcal{L}_t$ because the only source of uncertainty within the economy is the uncertain time of death T. It's worth highlighting that there is no market agents can trade in before they are born. This restriction introduces an incompleteness in the overlapping generations model (OLG) model, manifesting itself in the endogenous determination of each agent's initial consumption. Specifically, we have shown in Equation (14) that

$$\beta = (\nu + \rho)\gamma\psi = (\nu + \rho)\gamma \int_{t}^{\infty} e^{-(r+\nu)u} \, du = \frac{\nu + \rho}{\nu + r} \gamma, \tag{84}$$

where the last equality follows from Equation (12) and the fact that $M_u = M_t e^{-r(u-t)}$. This is the first equation the risk-free rate r and the initial consumption per unity of output β have to satisfy. Equation (16), which we prove next, is the second equation that r and β have to satisfy. We verify Equation (15) by combining the FOC for optimal consumption with the aggregate resource constraint

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t} \, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \beta Y_s e^{-\rho(t-s)} \frac{M_s}{M_t} \, ds. \tag{85}$$

We multiply both sides of Equation (85) by the strictly positive discount factor M_t and get

$$M_t Y_t = \int_{-\infty}^{t} \nu e^{-(\nu+\rho)(t-s)} \beta Y_s M_s \, ds. \tag{86}$$

We use the dynamics of the DF, $dM/M = -rdt$, and the dynamics of aggregate output, $dY/Y = \mu_Y dt$, to determine the dynamics of the left hand side of Equation (86). Specifically,

$$\frac{dM_t Y_t}{M_t Y_t} = \frac{dM_t}{M_t} + \frac{dY_t}{Y_t} = (\mu_Y - r)dt. \tag{87}$$

We differentiate the right side of Equation (86) and emphasize that we integrate over the birth cohort variable s and subsequently use Leibniz rule to determine the derivative of this integral w.r.t. time t. Hence,

$$\begin{aligned} d \int_{-\infty}^{t} \nu e^{-(\nu+\rho)(t-s)} \beta Y_s M_s \, ds &= \left(-(\nu + \rho) \int_{-\infty}^{t} \nu e^{-(\nu+\rho)(t-s)} \beta Y_s M_s \, ds + \nu \beta Y_t M_t \right) dt \\ &= -\left(\rho + \nu\left(1 - \beta\right)\right) Y_t M_t \, dt. \end{aligned} \tag{88}$$

In order for Equation (86) to hold the drift of the left hand side given in Equation (87) and the drift of the right hand side given in Equation (88) have to be the same. Equating both drifts leads to

$$\mu_Y - r = -\left(\rho + \nu\left(1 - \beta\right)\right). \tag{89}$$

Solving Equation (89) for the risk-free rate r_t leads to

$$r = \rho + \mu_Y + \nu(1-\beta) = \rho + \nu(\gamma - \beta), \tag{90}$$

where we obtain the second equality in Equation (90) by using Equation (4) for growth μ_Y. This verifies Equation (16) of the main text and provides the second equation that β and r have to satisfy. Plugging in for β given in Equation (84) into Equation (90) leads to

$$r = \rho + \nu\left(\gamma - \frac{\nu+\rho}{\nu+r}\gamma\right) = \rho + \frac{\nu\gamma}{\nu+r}\left(r-\rho\right). \tag{91}$$

Simplyfing Equation (91) and using the fact that growth is $\mu_Y = \nu(\gamma-1)$ leads to the quadratic equation $(r-\rho)(r-\mu_Y) = 0$ with the two solutions $r = \rho$ and $r = \mu_Y$. We rule out the second solution because it leads to a bubble. Plugging in for $r = \rho$ into Equation (84) leads to $\beta = \gamma$.

To prove Equation (17) for the wealth-output ratio, we compute total wealth in the economy by integrating over the financial wealth of all agents currently alive. Specifically,

$$\begin{aligned} W_t &= \int_{-\infty}^{t} \nu e^{-\nu(t-s)} W_{s,t}\, ds = \frac{1}{\nu+\rho}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t}\, ds \\ &= \frac{1}{\nu+\rho}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \beta_s Y_s e^{-\rho(t-s)}\frac{M_s}{M_t}\, ds = \frac{1}{\nu+\rho}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma Y_s e^{(r-\rho)(t-s)}\, ds \\ &= \frac{1}{\nu+\rho}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t}\, ds = \frac{Y_t}{\nu+\rho} = \psi Y_t, \end{aligned} \tag{92}$$

We obtain the second equality by using Equation (13), the third equality by using Equation (10) and $C_{s,s} = \beta_s Y_s$, the forth equality by using the solution γ for β, the fifth equality by using the definition of individual endowments $Y_{s,t} = \gamma Y_s$, the sixth equality by using the resource constraint given in Equation (2), and the last equality by plugging in for the price of the life annuity $\psi = 1/(\nu+\rho)$.

The total wealth in the economy is equal to $W_t = \psi Y_t$ and thus the price-dividend ratio of the market portfolio is $\psi = 1/(\nu+\rho)$ since the market portfolio is defined as a claim on the total wealth of all agents currently alive. Plugging the risk-free rate $r = \rho$ into optimal demand for consumption leads to optimal consumption in Equilibrium, that is,

$$C_{s,t} = C_{s,s}e^{(r-\rho)(t-s)} = C_{s,s}, \qquad \forall\, s \le t \le T. \tag{93}$$

Hence, there is no individual consumption growth in the economy and there is no consumption inequality within a birth cohort. Plugging in for $\beta = \gamma$ leads to

$$C_{s,t} = \beta_s Y_s = \gamma Y_s = Y_{s,t}, \qquad \forall\, s \le t \le T. \tag{94}$$

This proves Equation (18). Hence, consumption growth across birth cohorts is equal to aggregate output growth since $\forall\, \Delta > 0$ we have that

$$\frac{1}{\Delta}\log\left(\frac{C_{s+\Delta,t}}{C_{s,t}}\right) = \frac{1}{\Delta}\log\left(\frac{\gamma Y_{s+\Delta}}{\gamma Y_s}\right) = \frac{1}{\Delta}\log\left(\frac{Y_{s+\Delta}}{Y_s}\right) = \mu_Y, \qquad s \le t \le T. \tag{95}$$

This proves Proposition 1.

Proof 2 (Proof of Proposition 2)**.** *There is no endowment heterogeneity and thus the time-t value of the endowment of an agent born at time s is $Y_{s,t} = Y_t$. The market is complete and thus each agent chooses consumption to maximize her expected utility given in Equation (7) subject to the static budget* *constraint given in Equation (8). Solving the first order condition of this maximization problem leads to the optimal path for consumption given in Equation (10). Inserting optimal consumption into the static budget condition leads to the optimal time-t total wealth of an agent born at time s given in Equation (13). To pin down optimal consumption, we still need to solve for initial consumption $C_{s,s}$ or, equivalently, the initial consumption per unit of output defined as $\beta_s = C_{s,s}/Y_s$. To determine β_s we value the endowment stream of an agent and then use the fact that agents consume a constant fraction out of their wealth (see Equation (13)).*

So far the derivations are the same as in our benchmark exogenous growth model with heterogeneity across birth cohorts. We now value the individual endowment stream $Y_{s,t} = Y_t$ and thus we have to deviate from the previous case. Specifically, the value of the endowment stream ($Y_{s,t} = Y_t$) of birth cohort s at time t is

$$H_{s,t} = E_t\left[\int_t^T \frac{M_u}{M_t} Y_{s,u}\, du\right] = \int_t^\infty e^{-(r+\nu)(u-t)} \frac{M_u}{M_t} Y_u\, du = Y_t\phi, \qquad \forall\, s \le t \le T, \tag{96}$$

where ϕ is the valuation ratio of a variable annuity that continuously pays aggregate output until time of death T. We use the aggregate resource constraint and Equation (13) to determine aggregate wealth. Specifically,

$$Y_t = \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t} ds = (\rho+\nu) \int_{-\infty}^t \nu e^{-\nu(t-s)} W_{s,t} ds = (\rho+\nu)\, W_t \tag{97}$$

where W_t denotes wealth of all agents currently alive. Hence, from Equation (97) follows that the wealth-consumption ratio is constant and given by

$$\phi \equiv \frac{W_t}{Y_t} = \frac{1}{\rho+\nu}. \tag{98}$$

Since every newborn agent receives an endowment stream that is the same as aggregate output, the initial wealth of a newborn is equal to the total wealth and thus $\beta_s = 1$ for all s. Specifically,

$$\beta_s = \frac{C_{s,s}}{Y_s} = (\nu+\rho)\frac{W_{s,s}}{Y_s} = (\nu+\rho)\frac{H_s}{Y_s} = (\nu+\rho)\frac{W_s}{Y_s} = 1. \tag{99}$$

We plug the optimal demand for consumption given in Equation (10) into the aggregate resource constraint given in Equation (2) and use the fact that $\beta_t = 1\, \forall t$ to get

$$M_t Y_t = \int_{-\infty}^t \nu e^{-(\nu+\rho)(t-s)} Y_s M_s\, ds \tag{100}$$

Similar to Proposition 1 we differentiate both sides of Equation (100) w.r.t. time t by applying Leibniz's integral rule and solve for the risk-free rate. Specifically, equating the drifts of both sides of Equation (100) leads to

$$\mu_Y - r = -\nu - \rho + \nu \qquad \Rightarrow \qquad r = \rho + \mu_Y. \tag{101}$$

The price of the life annuity is

$$\psi = E_t\left[\int_t^T \frac{M_u}{M_t}\, du\right] = \int_t^\infty e^{-(\nu+r)(u-t)}\, du = \frac{1}{\nu+r} = \frac{1}{\nu+\rho+\mu_Y}. \tag{102}$$

Plugging the risk-free rate $r = \rho+\mu_Y$ into optimal demand for consumption leads to optimal consumption *in equilibrium, that is,*

$$C_{s,t} = C_{s,s}e^{(r-\rho)(t-s)} = C_{s,s}e^{\mu_Y(t-s)} = Y_s e^{\mu_Y(t-s)} = Y_t, \qquad \forall\, s \le t \le T. \tag{103}$$

Hence, individual consumption growths at the same rate as the economy. This proves Proposition 2.

A.2 Endogenous growth

We first provide more specific details regarding the theoretical arguments, and thereafter we will present the proofs of Propositions 3 and 4. The endogenous growth model described in Section 3 can be thought of a two stage decision-making process for each agent within a birth cohort: the ex-ante and the ex-post stages. The ex-ante stage occurs before the agent makes her decision to innovate. Here, agents compare the utility from not innovating to the expected utility from innovating. Following the ex-ante stage is the ex-post stage. This stage occurs after the decision to innovate has been made and the results of that decision—whether successful or unsuccessful—are known. At this point, all uncertainty related to the innovation process has been resolved. At this stage every agent solves a static consumption-investment problem factoring in the different circumstances determined by the innovation outcomes. Specifically, they maximize expected utility given in Equation (29) subject to the static budget constraint given in Equation (30), where the latter depends on their respective endowment streams, which could be the endowment stream Y_s, the augmented endowment stream $A\delta Y_s$ due to successful innovation, or the reduced endowment stream δY_s due to failed innovation. This leads to an expected utility level for the three different types—successful innovators, unsuccessful innovators, and non innovators—that agents maximize over in the ex-ante stage. Importantly, the optimal ex-ante and ex-post decision is the same for each birth cohort but depends on the agent's type, $(h, \Delta) \in \{1, \ldots, H\} \times [1, \bar{\Delta}]$, at the ex-ante stage and the type, $i = \iota(h, \Delta) \in \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$, at the ex-post stage, in which the latter type is the result of the innovation decision that we capture by the function $\iota(h, \Delta)$. Before we solve this two stage decision problem we emphasize that the second stage is a dynamic problem that leads to the optimal consumption process,$\{C^i_{s,t}\}_{\{s \le t \le T\}}$, that can be solved as a static optimization problem which we discuss in detail in Section 2 and that was originally developed in Cox and Huang (1989) and Karatzas and Shreve (1998).

We solve the model by backward induction and start with summarizing the derivation of the ex-post stage provided in the main text before we provide the results of the ex-ante stage in the proof of Proposition 3. Specifically, we identify a type (h, Δ) agent with the index $i = \iota(h, \Delta) \in \mathcal{I}$ because we know the realization of the innovation decision. The FOC for the static optimization problem leads to the optimal consumption process given in Equation (31). The consumption process $C^i_{s,t}$ depends on initial consumption $C^i_{s,s}$, which due to log utility equals $(\rho + \nu)W^i_{s,s}$. Moreover, we can derive initial wealth from the static budget constraint given in Equation (30). Specifically,

$$W^i_{s,s} = E_s\left[\int_s^T \frac{M_t}{M_s} Y^i_{s,t}\, dt\right] = \int_s^\infty e^{-(\nu+r)(t-s)} Y^i_{s,t}\, dt = \int_s^\infty e^{-(\nu+r)(t-s)} \omega_i Y_s\, dt = \omega_i Y_s \psi. \tag{104}$$

The valuation ratio is equal to the price of a life annuity, $\psi = 1/(\nu + r)$, given in Equation (12) because the endowment stream is constant over time for all $i \in \mathcal{I} = \{\text{se}, \text{ue}, \text{ne}\}$. There are three different levels for the endowment streams described by ω_i that is given in Equation (21). Plugging optimal consumption $C^i_{s,t}$ into expected utility given in Equation (29) and substituting $(\rho + \nu)\omega_i Y_s \psi$ for initial consumption $C^i_{s,s}$ leads to

$$U^i_s = \frac{\log\left(\omega^i\right)}{\rho + \nu} + \bar{U}_s \tag{105}$$

after some algebra with $\bar{U}_s$ given in Equation (33). This concludes the derivation of the ex-post stage.

Proof 3 (Proof of Proposition 3)**.** *In the ex-ante stage an agent of type (h, Δ) decides whether to become an entrepreneur or not. If she does not innovate, then, $\iota(h, \Delta) = ne$, and her lifetime expected utility is*

$$EU_s^{ne} \equiv U_s^{ne} = \frac{\log(1)}{\rho + \nu} + \bar{U}_s. \tag{106}$$

If she decides to innovate and is successful, then, $\iota(h, \Delta) = se$, and her lifetime expected utility is

$$U_s^{se} = \frac{\log(A\delta)}{\rho + \nu} + \bar{U}_s. \tag{107}$$

If she decides to innovate and is unsuccessful, then, $\iota(h, \Delta) = ue$, and her lifetime expected utility is

$$U_s^{ue} = \frac{\log(\delta)}{\rho + \nu} + \bar{U}_s. \tag{108}$$

The perceived success probability of a type (h, Δ) agent is Δ/H and thus her expected utility from innovating is

$$EU_s^{e} \equiv \frac{\Delta}{H} U_s^{se} + \left(1 - \frac{\Delta}{H}\right) U_s^{ue} = \frac{\Delta}{H} \frac{\log(A\delta)}{\rho + \nu} + \left(1 - \frac{\Delta}{H}\right) \frac{\log(\delta)}{\rho + \nu} + \bar{U}_s. \tag{109}$$

Hence, it is optimal for agent (h, Δ) to innovate if here expected utility from innovating exceeds the utility from not innovating, that is, $EU_s^e > EU_s^{ne}$. Let Δ^*/H denote the threshold probability for which an agent is indifferent between innovating and not innovating. We have that

$$\frac{\Delta^*}{H} \log(A\delta) + \left(1 - \frac{\Delta^*}{H}\right) \log(\delta) = \log(1) \qquad \Rightarrow \qquad \Delta^* = -H \frac{\log\ \delta}{\log A} \geq 0. \tag{110}$$

Hence, a type (h, Δ) agent innovates if $\Delta > \Delta^*$ and does not innovate if $\Delta \leq \Delta^*$. This concludes the derivation of the ex-ante stage and thus the derivation of the agent's decision problem.

The threshold probability Δ^*/H is the same for each of the H different strategies to innovate and only one of the strategies leads to a successful innovation., Hence, the probability that there is innovation in equilibrium is $\alpha = \mathbb{P}(\Delta > \Delta^*)$. The parameter Δ is uniformly distributed on the interval $[1, \bar{\Delta}]$. Moreover, there is no innovation if $\Delta^* \leq 1$ and there is always innovation if $\Delta^* \geq \bar{\Delta}$. If $1 < \Delta^* < \bar{\Delta}$, then

$$\alpha = \mathbb{P}(\Delta > \Delta^*) = 1 - \mathbb{P}(\Delta \leq \Delta^*) = 1 - \frac{\Delta^* - 1}{\bar{\Delta} - 1} = \frac{\bar{\Delta} + H \frac{\log(\delta)}{\log(A)}}{\bar{\Delta} - 1} = \frac{1 + \mathcal{D} + H \frac{\log(\delta)}{\log(A)}}{\mathcal{D}}, \tag{111}$$

where $\mathcal{D} = \bar{\Delta} - 1$ denotes the belief diversity index. The success probability α is strictly increasing if $1 < \Delta^* < \bar{\Delta}$ because

$$\frac{\partial \alpha(\mathcal{D})}{\partial \mathcal{D}} = \frac{1 - \alpha(\mathcal{D})}{\mathcal{D}} > 0. \tag{112}$$

Hence, α is weakly increasing for all Δ^*. There is a continuum of investors for each of the H different strategies to innovate and thus the equilibrium innovation probability coincides with the ex-post fraction of innovators in equilibrium. Only one of the H possible strategies leads to success. Hence, we have in equilibrium that for every birth cohort s the fraction α/H is successful and has total wealth $A\delta\psi Y_s$, the fraction $\alpha(1 - 1/H)$ is unsuccessful and has total wealth $\delta\psi Y_s$, and the fraction $(1 - \alpha)$ does not innovate and has total wealth ψY_s.

Proof 4 (Proof of Proposition 4)**.** *We know that for every birth cohort there are three types of agents after the innovation uncertainty is resolved. Specifically, (i) the fraction α/H of successful innovators with endowment stream $A\delta Y_s$, (ii) the fraction $\alpha(1-1/H)$ of unsuccessful innovators with endowment stream δY_s, and the fraction $(1-\alpha)$ of non-innovators with endowment stream Y_s. Hence, we have that $Y^i_{s,t} = \omega^i Y_s$ and λ^i_s is given in Equation (39). It follows that*

$$\sum_{i\in\mathcal{I}} \lambda^i_s Y^i_{s,t} = \sum_{i\in\mathcal{I}} \lambda^i_s \omega^i Y_s = \gamma Y_s \tag{113}$$

with γ given in Equation (40). Plugging into the aggregate resource constraint leads to

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma Y_s \, ds = Y_t, \tag{114}$$

Following the same arguments as in Section 2 and differentiating both sides of Equation (114) w.r.t. time t (by applying Leibniz's integral rule to the lhs) leads to to the equilibrium aggregate output growth rate $\mu_Y = \nu(\gamma-1) = \nu\alpha NCF$ with NCF given in Equation (41).

The first derivative of economic growth with respect to belief diversity $\mathcal{D}$ is

$$\frac{\partial \mu_Y(\mathcal{D})}{\partial \mathcal{D}} = \frac{\nu NCF \partial\alpha(\mathcal{D})}{\partial \mathcal{D}} = \frac{\nu\left(1-\alpha(\mathcal{D})\right) NCF}{\mathcal{D}}. \tag{115}$$

If NCF is positive, then economic growth is weakly increasing in diversity and the opposite is true if NCF is negative.

A.3 Asset Pricing and Inequality

Proof 5 (Proof of Proposition 5)**.** *The derivation of the equilibrium interest rate is similar to the exogenous growth case discussed in Propositions 1 and 2 of Section 2. The difference to the exogenous growth case is that there is also heterogeneity within a birth cohort, that is, there are agents that do not innovate and agents that innovate are either successful or not. We know from Equation (31) that the FOC of a type $i=(h,\Delta)$ agent is*

$$C^i_{s,t} = C^i_{s,s} e^{-\rho(t-s)} \frac{M_s}{M_t}. \tag{116}$$

Plugging consumption $C_{s,t}$ into the aggregate resource constraint leads to

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\sum_{i\in I} \lambda^i_s C^i_{s,t}\right) ds = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \left(\sum_{i\in I} \lambda^i_s C^i_{s,s}\right) \frac{M_s}{M_t}\, ds. \tag{117}$$

Define average consumption of the newborns relative to the economy wide average, that is,

$$\beta_s = \frac{\sum_{i\in I} \lambda^i_s C^i_{s,s}}{Y_s} = \beta \qquad \forall s. \tag{118}$$

We verify below that β is constant. Hence, we get

$$Y_t M_t = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \beta Y_s M_s \, ds. \tag{119}$$

after multiplying with M_t. Equation (119) is identical to Equation (15) and thus following the same steps as in the proof of Proposition 1 we get

$$\frac{dM_t}{M_t} = -r\,dt, \qquad \text{with} \qquad r = \rho + \mu_Y + \nu(1-\beta). \tag{120}$$

The beta differs from the beta in the exogenous growth case which we compute next. Due to log utility we have that

$$\beta^i = \frac{C^i_{s,s}}{Y_s} = (\nu+\rho)\frac{W^i_{s,s}}{Y_s} = \omega_i(\nu+\rho)\psi, \tag{121}$$

where the last equality follows from Equation (30). Plugging β^i into Equation (118) and plugging in for λ^i_s given in Equation (38) leads to

$$\beta = \frac{\sum_{i\in I}\lambda^i_s C^i_{s,s}}{Y_s} = \gamma(\nu+\rho)\psi \tag{122}$$

Plugging in for endogenous growth in Equation (120) leads to

$$r = \rho + \nu(\gamma-1) + \nu(1-\beta) = \rho + \nu(\gamma-\beta). \tag{123}$$

Hence, we have two equations that r and β have to simultaneously satisfy. Suppose both r and β are constant. Then the price of the life annuity is

$$\psi = E_t\left[\int_t^T \frac{M_u}{M_t}\,du\right] = \int_t^\infty e^{-(\nu+r)(u-t)}\,du = \frac{1}{\nu+r}. \tag{124}$$

Hence, $\beta = \gamma(\nu+\rho)/(\nu+r)$ and we have that

$$r = \rho + \nu(\gamma-\beta) = \rho + \nu\gamma\frac{r-\rho}{\nu+r} \quad\Leftrightarrow\quad r^2 - (\mu_Y+\rho)r + \mu_Y\rho = 0 \tag{125}$$

There are two solutions to this quadratic equation $r=\rho$ and $r=\mu_Y$. We exlude the second solution because it leads to a bubble. Plugging $r=\rho$ into the price of life annuity leads to $\psi = 1/(\nu+\rho)$. We plug the solution for r into Equation (122) to get β, that is,

$$\beta = \gamma = 1 + \alpha NCF. \tag{126}$$

Total wealth of all agents currently alive is

$$\begin{aligned}
W_t &= \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\sum_{i\in I} \lambda_s^i W_{s,t}^i \right) ds \\
&= \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\sum_{i\in I} \lambda_s^i C_{s,t}^i \right) ds \\
&= \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\sum_{i\in I} \lambda_s^i C_{s,s}^i \right) e^{-\rho(t-s)} \frac{M_s}{M_t}\, ds \\
&= \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \beta_s Y_s e^{-\rho(t-s)} \frac{M_s}{M_t}\, ds \\
&= \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma Y_s e^{(r-\rho)(t-s)}\, ds \\
&= \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t}\, ds = \frac{Y_t}{\nu+\rho}.
\end{aligned} \tag{127}$$

Hence, the price-dividend ratio is $\psi = 1/(\nu+\rho)$.

Proof 6 (Proof of Proposition 6)**.** *Let $NCF > 0$. Before we determine the variance of $\beta_{s,t}^i$, we verify that average $\beta_{s,t}^i$ in equilibrium is one. Specifically,*

$$\begin{aligned}
\mathrm{E}\left[\beta_{s,t}^i\right] &= \int_{-\infty}^{t} \left(\sum_{i\in I} \lambda_s^i \beta_{s,t}^i \right) \nu e^{-\nu(t-s)}\, ds = \int_{-\infty}^{t} \left(\sum_{i\in I} \lambda_s^i \omega^i e^{-\mu_Y(t-s)} \right) \nu e^{-\nu(t-s)}\, ds \\
&= \int_{-\infty}^{t} \gamma \nu e^{-(\nu+\mu_Y)(t-s)}\, ds = \frac{\gamma\nu}{\nu+\mu_Y} = \frac{\gamma\nu}{\nu+\nu(\gamma-1)} = 1.
\end{aligned} \tag{128}$$

We obtain the second equality by plugging in the value for $\beta_{s,t}^i$ given in Equation (49), the third equality by plugging in the value for $\sum_{i\in I} \lambda_s^i \omega^i$ which is γ and was derived in Equations (38)-(40), the fourth equality by computing, the integral, and the fifth equality by plugging in the value for μ_Y given in Equation (42). The variance of $\beta_{s,t}^i$ is

$$\begin{aligned}
\mathrm{Var}\left[\beta_{s,t}^i\right] &= \mathrm{E}\left[\left(\beta_{s,t}^i\right)^2\right] - \mathrm{E}\left[\beta_{s,t}^i\right]^2 = \mathrm{E}\left[\left(\beta_{s,t}^i\right)^2\right] - 1 \\
&= \int_{-\infty}^{t} e^{-2\mu_Y(t-s)} \left(\sum_{i\in I} \omega_i^2 \lambda_s^i \right) \nu e^{-\nu(t-s)}\, ds - 1 \\
&= \left(A^2\delta^2 \frac{\alpha}{H} + \delta^2\alpha\left(1 - \frac{1}{H}\right) + (1-\alpha) \right) \frac{\nu}{\nu + 2\mu_Y} - 1.
\end{aligned} \tag{129}$$

We obtain the third equality by plugging in the value for $\beta_{s,t}^i$ and the fourth equality by plugging in the value for ω^i given in Equation (21), the value for λ_i given in Equation (39) and deriving the sum. The remaining steps are straightforward algebra and plugging in the value for μ_Y given in Equation

(42). Specifically,

$$
\begin{aligned}
\mathbb{V}ar\left[\beta_{s,t}^i\right] &= \alpha\left(\left(\frac{A^2}{H}+\ \left(1-\frac{1}{H}\right)\right)\delta^2-1\right)\frac{\nu}{\nu+2\mu_Y}+\frac{\nu}{\nu+2\mu_Y}-1 \\
&= \frac{\alpha}{1+2\alpha NCF}\left(\left(\frac{A^2}{H}+\ \left(1-\frac{1}{H}\right)\right)\delta^2-1\right)+\frac{1}{1+2\alpha NCF}-1 \qquad (130)\\
&= \frac{\alpha}{1+2\alpha NCF}\left(\left(\frac{A^2}{H}+\ \left(1-\frac{1}{H}\right)\right)\delta^2-2NCF\right).
\end{aligned}
$$

The variance is positive and thus the term inside the bracket of Equation (130), which is independent of disagreement is positive. The first term is strictly increasing in α and thus consumption inequality is strictly increasing in α. We know from Proposition 3 that α is weakly increasing in disagreement and thus consumption inequality is weakly increasing in disagreement. We know from Proposition 4 that economic growth is weakly increasing in disagreement if NCF is positive. Hence, inequality is weakly increasing in economic growth.

Proof 7 (Proof of Proposition 7)**.** *The lifetime expected utility of an entrepreneur based on the success probability of $1/H$ is*

$$EU_s^{e,0}=\frac{1}{H}U_s^{se}+\left(1-\frac{1}{H}\right)U_s^{ue}=\frac{1}{\nu+\rho}\left(\frac{1}{H}\log(A)+\log(\delta)\right)+\bar{U}_s, \qquad (131)$$

where U_s^{se} and U_s^{ue} are given in Equations (107) and (108), respectively. The lifetime utility from not innovating is given in Equation (106), that is, $U_s^{se}=\bar{U}_s$. Hence,

$$EU_s^{e,0}-U_s^{se}=\frac{1}{\nu+\rho}\left(\frac{1}{H}\log(A)+\log(\delta)\right). \qquad (132)$$

A.4 Entrepreneurial risk sharing

In order to provide additional details of the theoretical arguments in Subsections 5.1.1, 5.1.2, 5.1.3, and 5.2 we first rewrite the utility given in Equation (30) as follows

$$U_s^i=\frac{\log\left(W_{s,s}^i\right)}{\rho+\nu}+\underbrace{\frac{\log\left(\rho+\nu\right)}{\rho+\nu}+\int_s^\infty e^{-(\rho+\nu)(t-s)}\log\left(\frac{M_s}{M_t}\right)\,dt}_{=\hat{U}_s}, \qquad (133)$$

where the second term of lifetime utility is independent of the agent's type. Following the same arguments as in Section 3 the probability Δ^*/H for which an agent is indifferent between innovating and not innovating has to satisfy

$$\frac{\Delta^*}{H}\log\left(W_{s,s}^{\mathrm{se}}\right)+\left(1-\frac{\Delta^*}{H}\right)\log\left(W_{s,s}^{\mathrm{ue}}\right)=\log\left(W_{s,s}^{\mathrm{ne}}\right). \qquad (134)$$

Solving this equation for Δ^* leads to

$$\Delta^*=\frac{\log\left(W_{s,s}^{\mathrm{ne}}/W_{s,s}^{\mathrm{ue}}\right)}{\log\left(W_{s,s}^{\mathrm{se}}/W_{s,s}^{\mathrm{ue}}\right)}. \qquad (135)$$

Plugging the wealth levels of the different tax examples given in Equations (54) and (62) leads to Δ_τ^* given in Equations (55) and (63), respectively. Similarly, plugging the wealth level from the venture capital example given in Equation (76) leads to Δ_θ^* given in Equation (77). If we allow for costly effort as in Section 5.1.3, then

$$\Delta^* = \frac{\log\left(W_{s,s}^{\text{ne}}/(W_{s,s}^{\text{ue}}/(1+e))\right)}{\log\left((W_{s,s}^{\text{se}}/(1+e))/W_{s,s}^{\text{ue}}/(1+e))\right)} = \frac{\log\left((1+e)W_{s,s}^{\text{ne}}/(W_{s,s}^{\text{ue}}\right)}{\log\left(W_{s,s}^{\text{se}})/W_{s,s}^{\text{ue}}\right)}. \tag{136}$$

Plugging in for the wealth levels given in Equation (71) leads to Δ_τ^* given in Equation (73). The likelihood of innovation is always equal to

$$\alpha_{\tau,e} = \mathbb{P}\left(\Delta > \Delta^*\right) = \frac{\bar{\Delta} - \Delta^*}{\bar{\Delta} - 1} \tag{137}$$

where Δ^* differs across the three tax and the venture capital scenarios. We know from the exogenous growth benchmark derived in Section 2 and the endogenous growth model in Section 3 that aggregate growth is given by

$$\mu_Y = \nu(\gamma - 1). \tag{138}$$

The same is true in this section in all three tax scenarios and the venture capital fund. However, the γ, which we derive by plugging in individual endowments in the aggregate resource constraint, is different in the four scenarios. Specifically, plugging for the individual endowments given in Equations (53), (61), (70), and (76) leads to

$$\gamma = \sum_{i\in\mathcal{I}} \lambda_s^i \frac{Y_{s,t}^i}{Y_s} = \begin{cases} \tau + (1-\tau)\sum_{i\in\mathcal{I}} \lambda_s^i\omega^i & \text{in Tax Scenario 1} \\ \tau(Y_t/Y_s) + (1-\tau)\sum_{i\in\mathcal{I}} \lambda_s^i\omega^i & \text{in Tax Scenario 2} \\ \tau + (1-\tau)\sum_{i\in\mathcal{I}} \lambda_s^i\omega^i & \text{in Tax Scenario 3} \\ \theta(1+\text{NCF}) + (1-\theta)\sum_{i\in\mathcal{I}} \lambda_s^i\omega^i & \text{in VC Scenario} \end{cases} \tag{139}$$

with

$$\begin{aligned} \sum_{i\in\mathcal{I}} \lambda_s^i\omega^i &= \frac{\alpha}{H}A\delta + \alpha\left(1 - \frac{1}{H}\right)\delta + (1-\alpha) \\ &= 1 + \alpha\left(\frac{1}{H}(A\delta - 1) - \left(1 - \frac{1}{H}\right)(1-\delta)\right) = 1 + \alpha\text{NCF}. \end{aligned} \tag{140}$$

The growth rate in Tax Scenario 1, 3, and the VC Scenario are given in Equations (59), (74), and (81). Moreover, the risk-free rate is $r = \rho$ in all three scenarios. The proof is identical to the sections 2 and 3 and thus omitted.

The last part we need to derive is the growth rate and risk-free rate in Tax scenario 2. The results are summarized in Proposition 8 which we prove next.

Proof 8 (Proof of Proposition 8)**.** *Recall that*

$$\gamma(s,t) = \tau(Y_t/Y_s) + (1-\tau)\sum_{i\in\mathcal{I}} \lambda_s^i\omega^i = \tau(Y_t/Y_s) + (1-\tau)\left(1 + \alpha_\tau(r,\mu_Y)\right) \tag{141}$$

Plugging back into the aggregate resource constraint leads to

$$Y_t = \int_{-\infty}^t \nu e^{-\nu(t-s)}\gamma(s,t)Y_s\,ds = \tau Y_t + (1-\tau)\left(1 + \alpha_\tau(r,\mu_Y)NCF\right)\int_{-\infty}^t \nu e^{-\nu(t-s)}Y_s\,ds. \tag{142}$$

Subtracting τY_t from the left hand side and dividing by $(1-\tau)$ leads to

$$Y_t = (1+\alpha_\tau(r,\mu_Y)NCF)\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_s\, ds. \tag{143}$$

This is the same equation that we got in Subsection 3.4. Hence, differentiating both sides w.r.t time t leads to the same equation for growth as in Subsection 3.4, that is, Equation (42), but with a different innovation likelihood $\alpha_\tau(\mu_Y, r)$. Specifically,

$$\mu_Y = \nu\alpha_\tau(r,\mu_Y)NCF \tag{144}$$

with NCF given in equation (41). Similar to Subsection 4.1 we determine the ratio of total consumption of newborns to output. Specifically, v

$$\begin{aligned}\beta_s &= \frac{\sum_{i\in I}\lambda_s^i C_{s,s}^i}{Y_s} = (\nu+\rho)\frac{\sum_{i\in I}\lambda_s^i W_{s,s}^i}{Y_s} \equiv \beta_\tau(r,\mu_Y) \\ &= (\nu+\rho)\left((1-\tau)\psi(r)(1+\alpha_\tau(r,\mu_Y)NCF)+\tau\phi(r,\mu_Y)\right).\end{aligned} \tag{145}$$

where the last equality follows from plugging in Equation (62) for the wealth of newborns and using the fact that

$$\sum_{i\in\mathcal{I}}\lambda_s^i\omega^i = 1+\alpha_\tau(r,\mu_Y).$$

Following the same steps as in the proof of Proposition 5 but with a different β leads to

$$r = \rho+\mu_Y+\nu(1-\beta_\tau(r,\mu_Y)). \tag{146}$$

This concludes the proof of this proposition.